Output feedback stabilisation of networked control systems via switched system approach

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Output feedback stabilisation problem of networked control systems with bounded packet loss is addressed. Sufficient conditions for output feedback stabilisation are derived by using a packet-loss-dependent Lyapunov function. Different from existing results, we present a design for packet-loss-dependent output feedback controllers for two types of packet-loss processes: one is an arbitrary packet-loss process, and the other is a Markovian packet-loss process. Several numerical examples and simulations are worked out to demonstrate the effectiveness of the proposed design techniques.

Keywords: networked control systems; stabilisation; output feedback; switched systems; packet losses

1. Introduction

Networked control systems (NCSs) are distributed systems in which sensors, controllers and actuators are connected through a shared networked channel. Compared with the traditional point-to-point wiring, the use of the communication channels provides a control system with many advantages such as lower costs of cables and power, simpler installation and maintenance of the whole system, and higher reliability. Therefore, NCSs have found many industrial applications, and typical examples are computer integrated manufacturing systems, large-scale distributed industrial processes, tele-operation and tele-control, etc. In recent years, NCSs have received increasing attention from researchers in the field of information and control (Wong and Brockett 1999; Lin, Zhai, and Antsaklis 2003; Walsh and Ye 2001; Walsh, Hong, and Bushnell 2002; Montestruque and Antsaklis 2003).

One of the major issues raised in NCSs is the unreliable transmission paths because of limited bandwidth and large amount of data packet transmitted over one line, which may result in transmission delays and data packet dropout. The latter is a potential source of instability and poor performance of NCSs. Therefore, it is very essential for the real industrial applications to construct a feedback controller by using the most fresh information to stabilise an NCS. To our best knowledge, for stability analysis and controller synthesis of NCSs, two effective approaches have been used to deal with controller synthesis problem of NCSs with bounded packet loss: one is the delayed system approach (Nilsson, Bernhardsson, and Wittenmark 1998; Yue, Wang, Chu, and Xie 2004) where NCSs were modelled as systems with bounded delays, and the other is the switched system approach (Yu et al. 2004; Ishii 2006; Xiong and Lam 2007) where NCSs were modelled as switched systems with several subsystems. In addition, for these two approaches, two types of packet-loss processes have been considered in many existing results: one regards the data packet dropout as an arbitrary packet-loss process (Yu et al. 2004; Xiong and Lam 2007; Zhang, Chen, and Chen 2007); the other regards it as a stochastic process (Nilsson et al. 1998; Ling and Lemmon 2003; Wu and Chen 2007). Moreover, other problems are also concerned in existing results. Zhang, Branicky, and Phillips (2001) modelled NCSs with data packet dropout as asynchronous dynamic systems, but the stability condition derived by them is in the form of bilinear matrix inequalities which are difficult to solve. The results on $L_p$ stability for general non-linear NCSs with disturbances and data packet dropouts were presented in Nesic and Teel (2004). In order to reduce the amount of data transmitted over the network, Xie and...
Wang (2005) presented a framework for stabilisation of NCSs with limited data rates by only transmitting a non-singular transform of the state vector over the network channel. In Ma and Zhao (2006), switching impulse was considered in order to reduce the error between theory and application for an NCS. Gupta, Hassibi, and Murray (2007) studied a class of NCSs with packet loss, and designed optimal linear quadratic Gaussian controllers by using a separation principle. Output feedback stabilisation problem of a class of non-linear NCSs with non-decreasing non-linearities and limited-capacity communication channels was discussed in Cheng and Savkin (2007).

By re-formulating the system into a mixed logical dynamical system, Zhang, Shi, Chen, and Huang (2005) proposed the controller design for NCSs regulated by a network data transmission scheme via the predictive control approach.

The advantage of the switched system approach is that the controllers can make full use of the previous information to stabilise NCSs when the current state/out measurements are not available from the network. Based on bounded packet loss, NCSs were modelled as a class of switched systems, thus many switched systems theories (Branicky 1998; Daafouz, Riedinger, and Jung 2002; Sun and Ge 2005) can be used to discuss stability analysis and control synthesis of NCSs. Yu et al. (2004) studied the stabilisation of NCSs with bounded packet loss via both state feedback and output feedback. Xiong and Lam (2007) generalised the results in Yu et al. (2004) by introducing a packet-loss-dependent Lyapunov function, in which sufficient conditions in the form of linear matrix inequalities (LMIs) for the state feedback stabilisation of NCSs have been derived. Based on the theories for the discrete-time switched systems, disturbance attenuation issues for a class of NCSs under uncertain access delay and packet loss effects were considered in Lin, Zhai, and Antsaklis (2003), and an $H_{\infty}$ control problem for a remotely controlled system over a shared network was discussed in Ishii (2006). Sufficient conditions for exponential stability of NCSs were derived in Mu, Chu, and Hao (2003) and Zhang and Yu (2007), where the plant was controlled alternately by an open loop controller and a closed loop controller. State and output feedback were considered in Montestruque and Antsaklis (2003), where necessary and sufficient conditions for exponential stability were derived in the form of simple eigenvalue tests of a well-structured test matrix constructed in terms of the update time $h$ and the parameters of the plant. However, none of the aforementioned results concerned the design of multiple controllers.

In the field of control, if a single controller fails to solve a control problem, multiple controllers might be used in the hope that the problem may be solved by switching among these controllers. However, it is noticed that most of the existing results concerned the design for a single controller, even when NCSs are modelled as switched systems. By taking the packet loss in the backward channel as bounded delays, packet-loss-dependent controllers were designed in Zhang et al. (2005) and Wu and Chen (2007), but neither of them have considered the output feedback stabilisation.

In this article, enlightened by all the above analysis, we study the stabilisation of NCSs with the bounded packet loss via output feedback, and present the design for packet-loss-dependent output feedback controllers which are advantageous over the normal feedback controllers designed in Yu et al. (2004) and Xiong and Lam (2007). Different from the recent result in Zhang and Yu (2007), where sensors, controllers and actuators are time-driven, we assume that sensors are time-driven and controllers and actuators are event-driven. One can see that the existence of the event-driven actuators will complicate the modelling. Another existing result on output feedback is Matveev and Savkin (2006). It proposed necessary and sufficient stabilising conditions for the NCSs with bandwidth communication constraints, but it did not study the problem of the controller design in details. Different from Matveev and Savkin (2006), based on the switched Lyapunov function theories, we propose the packet-loss-dependent stabilising conditions in the form of LMIs and we can obtain stabilising controllers easily by solving these LMIs. More specifically, in this article, two types of packet-loss processes are considered: one is the arbitrary packet-loss process, and the other is the Markovian packet-loss process. For both cases, we derive packet-loss-dependent sufficient conditions in the form of LMIs for the stabilisation by using a packet-loss-dependent Lyapunov function, and propose the design for packet-loss-dependent output feedback controllers by solving some LMIs. Furthermore, for the NCSs in which the packet loss occurs only in the channel between the sensor and the controller, we derive the sufficient conditions for stabilisation of such NCSs via dynamic output feedback.

This article is organised as follows. Section 2 introduces the mathematical model of NCSs under study, and some definitions and lemmas are also presented in this section. Section 3 deals with the stabilisation problem of NCSs with the arbitrary packet-loss process, and stabilising output feedback controllers are constructed by using the feasible solutions of some LMIs. Section 4 discusses the stabilisation problem for NCSs with the Markovian packet-loss process, and stabilising
output feedback controllers are constructed by solving some LMIs. Section 5 contains an application of the design technique to the case of dynamic output feedback. Some numerical examples and simulations demonstrating the effectiveness of the design techniques are worked out in Section 6. Finally, the conclusions are provided in Section 7.

Notations. Throughout this article, the following notations are used. N denotes the set of all non-negative integer; for any two positive integers \( m \) and \( n \) satisfying \( n \geq m \), \([m, n] = \{m, m+1, \ldots, n\} \). Furthermore, denote \([m, n] \times [k, l] = \{(i, j) : i \in [m, n], j \in [k, l]\}\).

2. Problem formulation

Consider the NCS with the packet loss in both the channel between the sensor and the controller (the backward channel) and the channel between the actuator and the controller (the forward channel) illustrated in Figure 1, where the sensor is clock-driven and the controller and the actuator are event-driven. We first consider the NCS setup with a clock-driven sensor, and both the controller and the actuator are combined into one event-driven node illustrated in Figure 2, that is, network communication occurs only from the sensor to the controller through a communication channel with finite bandwidth. The NCS consisting of a discrete plant and a time-varying output controller can be described as

\[
x(t + 1) = Ax(t) + Bu(t),
\]

\[
y(t) = Cx(t),
\]

\[
u(t) = F(t)y(t),
\]

where \( t \in \mathbb{N} \), \( x(t) \in \mathbb{R}^n \) is the plant state vector, \( u(t) \in \mathbb{R}^m \) is the plant input vector and \( y(t) \in \mathbb{R}^p \) is the output of the plant. \( A, B \) and \( C \) are known real constant matrices with proper dimensions.

The piecewise continuous matrix function \( F(t) \) is the output feedback gain matrix to be designed. \( \bar{y}(t) \in \mathbb{R}^p \) is the output measurement that is successfully transmitted over the network. Thus, \( \bar{y}(t) \in \mathbb{R}^p \) is defined as

\[
\bar{y}(t) = \begin{cases} 
y(t), & \text{if the packet containing } y(t) \\
y(t - 1), & \text{otherwise}
\end{cases}
\]

We suppose that a sensor data containing the output information will substitute the old data when it is successfully sent to the controller through the communication channel, and the updated data is denoted by \( \bar{y}(t) \). The controller reads out the content of \( \bar{y}(t) \) and utilises it to compute the new control input, which will be applied to the plant. Furthermore, we suppose that the update instants of \( \bar{y}(t) \) is numerable and the set of successive update instants \( \{t_0 = 0, t_1, \ldots, t_k, \ldots\} \) is a subset of \( \mathbb{N} \). In what follows, we describe our modelling using the switched system approach.

Without loss of generality, we assume that the packet containing \( y(0) \) is transmitted to the controller successfully, that is, \( \bar{y}(0) = y(0) \), then \( x(1) = (A + BF(0)C)x(0) \). In the following time instant, if the data packet containing \( y(1) \) is transmitted to the controller successfully, then \( x(2) = (A + BF(1))x(1) \); otherwise, \( x(2) = Ax(1) + BF(1)Cx(0) = (A(A + BF(0)C) + BF(1)C)x(0) \).

We refer to the time interval between \( t_k \) and \( t_{k+1} \) as one transmission interval. In this pattern of transmission, the states of NCS (1) at the update instants can be described as follows:

\[
x(t_{k+1}) = \left( A^{t_{k+1}-t_k} + \sum_{j=0}^{t_{k+1}-t_k-1} A^jBF(t_{k+1} - l - 1)C \right)x(t_k),
\]

\[\times k \in \mathbb{N}.
\]

Define \( z(0) = x(0), \ z(1) = x(t_1), \ldots, z(k) = x(t_k) \), and

\[A_k = A^{t_{k+1}-t_k} + \sum_{j=0}^{t_{k+1}-t_k-1} A^jBF(t_{k+1} - l - 1)C.
\]

It follows that \( z(k + 1) = A_kz(k) \).
Without loss of generality, we assume that the maximum transmission interval is \( d \), and then the upper bound of the dropped data packets is \( d - 1 \). With a set of candidate gains \( \{ F_1, F_2, \ldots, F_d \} \) to be designed, we give the following feedback schedule algorithm to stabilise NCS (1).

**Schedule Algorithm 1:** For any \( t_k \), we assume that there is a counter along with the controller, which records the length of the last transmission interval \([t_{k-1}, t_k)\), and then take the packet-loss-dependent feedback gain as \( F_{t_k-t_{k-1}} \) in the following transmission interval \([t_k, t_{k+1})\), i.e. \( u(t) = F_{t_k-t_{k-1}} \tilde{y}(t), \ t \in [t_k, t_{k+1}) \).

Now, we let \( r(k) = t_{k+1} - t_k \), \( r(k) = i \), \( r(k-1) = j \), and denote

\[
\tilde{A}_{ij} = A^i + \sum_{l=0}^{j-1} A^l BF_j C.
\]

Thus, based on Schedule Algorithm 1, it is obvious that the evolution of NCS (1) at the successive update instants can be described as the following switched system:

\[
z(k + 1) = \tilde{A}_{\eta(k)} z(k), \ \ k \in \mathbb{N},
\]

where \( \tilde{A}_{\eta(k)} = A^{r(k)} + \sum_{l=0}^{r(k)-1} A^l BF_{r(k)-l} C \in \tilde{\Omega} = \{ A_{11}, A_{12}, \ldots, A_{jd}, A_{d1}, A_{d2}, \ldots, A_{dd} \} \), and for all \( k > 1 \), \( \eta(k) = (r(k), r(k-1)) \in [1, d] \times [1, d] \), with \( \eta(1) = (r(1), 1) \) which means that \( y(0) \) is transmitted to the controller successfully.

Now, we study mathematical modelling for an NCS with packet losses in both the forward and backward channels illustrated in Figure 1. Control input \( u(t) \) is defined as

\[
u(t) = \begin{cases} 
\tilde{u}(t), & \text{if the packet containing } \tilde{u}(t) \\
\tilde{u}(t-1), & \text{otherwise}.
\end{cases}
\]

We denote \( \{ t_0 = 0, t_1, \ldots, t_k, \ldots \} \) as the set of successive instants when packet \( y(t_k) \) is transmitted from the plant to the controller and then to the plant successfully and \( \{ t_{k-1}, t_k \} \) as a transmission interval. To design multiple controllers we assume that there is a counter along with the controller, which denotes the number of the packet loss successively at a transmission interval. Then, at the next transmission interval, the number would be read in by the controller to design new control input. The existence of such counter can be guaranteed by an acknowledgement (ACK) signal which is sent by the actuator to the controller to tell the acknowledgement of received packets at each sample instant. Notice that the information of packet losses in the forward channel at present time is not available when we design the controllers. Thus, we present the following schedule algorithm.

**Schedule Algorithm 2:** For any \( t_k \), we assume that there is a counter along with the controller, which records the length of the last transmission interval \([t_{k-1}, t_k)\), and then take the packet-loss-dependent feedback law \( u(t) = F_{t_k-t_{k-1}} \tilde{y}(t), \ t \in [t_k + 1, t_{k+1}) \).

Similarly, applying Schedule Algorithm 2 to NCS (1), and denoting \( z(k) = x(t_k) \) and \( \tilde{A}_{(k)} = A^{r(k)} + A^{r(k)-1} BF_{r(k)-2} C + \sum_{l=0}^{r(k)-3} A^l BF_{r(k)-l} C \) we obtain the following switched system:

\[
z(k + 1) = \tilde{A}_{(k)} z(k).
\]

Up to now, for general NCSs with packet-loss links, we present new mathematical models by introducing novel design methods.

**Remark 1:** Let \( F_1 = F_2 = \cdots = F_d = F \). Then our problem would become normal output feedback stabilisation problem discussed in Yu et al. (2004). In fact, if \( F_1 = F_2 = \cdots = F_d = F \), Equation (3) becomes \( \tilde{A} = A^i + \sum_{l=0}^{i-1} A^l BF_i C \). It follows that (4) reduces to system equation (15) in Yu et al. (2004). Thus, the output stabilisation problem discussed in Yu et al. (2004) is a special case of ours.

**Remark 2:** Here, we ignore the effect of transmission delays. However, all results can be generalised easily to NCSs with constant transmission delay \( \tau \), which is described as \( x(t+1) = Ax(t) + Bu(t-\tau), \) since \( \tau \) is always known. For an NCS with time-varying transmission delays, its stabilisation via multiple controllers will be addressed in the further.

Now, we present the following definitions and technical lemmas for later use.

**Definition 2.1** (Xiong and Lam 2007): A packet-loss process \( \{ r(t_k) \in \mathbb{N} : r(t_k) = t_{k+1} - t_k \} \) is said to be arbitrary if it takes values in the interval \([1, d]\) arbitrarily.

**Definition 2.2:** A packet-loss process \( \{ r(t_k) \in \mathbb{N} : r(t_k) = t_{k+1} - t_k \} \) is said to be Markovian if it is a Markov chain, where the transition probability matrix \( P = [p_{ij}] \in \mathbb{R}^{d \times d} \), with \( p_{ij} = \text{Pr}(r(t_{k+1}) = i| r(t_k) = j) \geq 0 \) for any \( (i, j) \in [1, d] \times [1, d] \), and \( \sum_{i=1}^{d} p_{ij} = 1 \).

**Lemma 2.3** (Boyd, Ghaoui, Feron, and Balakrishnan 1994): Given the symmetric matrix \( S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} \), where \( S_{11} \) is \( r \times r \), then the following three statements are equivalent:

(i) \( S_{11} < 0 \);
(ii) \( S_{22} - S_{12} S_{11}^{-1} S_{12} < 0 \);
(iii) \( S_{22} < 0, \ S_{11} - S_{12} S_{22}^{-1} S_{12} < 0 \).
Lemma 2.4 (Ho and Lu 2003): For a given $C \in \mathbb{R}^{p \times n}$ with $\text{rank}(C) = p$, assume that the singular value decomposition of $C$ is $C = MC_0N^T$, where $M \in \mathbb{R}^{p \times p}$ and $N \in \mathbb{R}^{n \times n}$ are unitary matrices and $C_0 \in \mathbb{R}^{p \times p}$ is a diagonal matrix with positive diagonal elements in decreasing order. $Q \in \mathbb{R}^{p \times n}$ is a symmetric matrix. Then there exists a matrix $Z \in \mathbb{R}^{p \times p}$ such that $CQ = ZC$ if and only if $Q = M[Q_1 \ 0 \ 0_2]N^T$, where $Q_1 \in \mathbb{R}^{p \times p}$, $Q_2 \in \mathbb{R}^{(n-p) \times (n-p)}$.

3. Stabilisation of NCSs with arbitrary packet-loss process

In this section, we suppose that the packet loss of NCS (1) is arbitrary. Sufficient conditions for the stabilisation via output feedback are derived via the switched system approach and stabilising controllers are designed by resolving some matrix inequalities.

Definition 3.1: A function $\phi : \mathbb{R}^n \to \mathbb{R}_+$ is of class $K$ if it is continuous, strictly increasing and $\phi(0) = 0$.

Without loss of generality, we assume that 0 is an unique equilibrium of NCS (1), and the state trajectory starts at $t_0 = 0$ with an initial state $x(0)$. The following result ensures the asymptotic stability of NCS (1). It is a generalisation of Lemma 1 in Yu et al. (2004).

Lemma 3.2: If there exists $q$ with $q \geq 1$ continuous functions $V_k : \mathbb{R}^n \to \mathbb{R}_+$ for all $k \in [1, q]$, and functions $\alpha$, $\beta$, $\gamma$ of class $K$ such that for all $x \in B_r = \{x : \|x\| \leq r\}$,

$$\alpha(\|x\|) \leq V_k(x) \leq \beta(\|x\|), \quad \forall i \in [1, q], \quad (6)$$

and

$$V_j(x(t_{k+1})) - V_j(x(t_k)) \leq -\gamma(\|x(t_k)\|), \quad \forall i, j \in [1, q], \quad (7)$$

for some time sequence $\{t_0 = 0, t_1, \ldots, t_k, \ldots\}$ which is a subset of $\mathbb{N}$, then NCS (1) is uniformly asymptotically stable.

Proof: The proof is essentially similar to that of Lemma 1 in Yu et al. (2004). Hence, many of the details are omitted. Taking the NCS illustrated in Figure 2, for example, we present some details.

In view of (2), for any $t \in (t_k, t_{k+1})$, we have the following inequality:

$$\|x(t)\| \leq \|A^t + \sum_{i=0}^{L-1} A^t B_i F_i C\| \|x(t_k)\| \leq c \|x(t_k)\| \quad (8)$$

where $c = \max_i \{\|A^t + \sum_{i=0}^{L-1} A^t B_i F_i C\|\}$.

Denote $V(x) = \sum_{i=1}^d \xi_i(x)V_i(x)$, where $\xi_i(x)$ is a piecewise continuous function with two values 0 and 1, and $\xi_i(x) = 1$ if $V(x) = V_i(x)$ and 0 otherwise.

Given any $\varepsilon > 0$, we let $\rho = \min\{\varepsilon, r\}$. In view of (6), there exists a positive number $\delta$ such that

$$\eta(\delta) = \sup_{\|x(t_0)\| \leq \delta} V(x) < \alpha(\rho/c) \leq \alpha(\varepsilon/c), \quad (9)$$

which leads to $\|x(t_0)\| \leq \delta$ implies that $\|x(t)\| \leq \varepsilon$, $\forall t \in \mathbb{N}$. Hence, we get NCS (1) is uniformly stable.

Furthermore, the uniformly asymptotical stability is guaranteed by the decreasing and positive properties of the sequence $V(x(t_k)) = \sum_{i=1}^d \xi_i(x(t_k))V_k(x(t_k))$ with $k \in \mathbb{N}$. Thus, this completes the proof. \hfill $\square$

Based on Lemma 3.2, we get the following sufficient condition for the stabilisation of NCS (1) illustrated in Figure 2 via output feedback.

Theorem 3.3: If there exist $d$ symmetric positive definite matrices $X_i$ and matrices $Y_i$, $Z_i$ with $i \in [1, d]$ such that

$$CX_i = Z_iC, \quad (10)$$

and

$$\begin{bmatrix}
X_j & \sum_{i=0}^{L-1} A^t B_i Y_i C \\
A^t X_j + \sum_{l=0}^{L-1} A^t B_i Y_i C & X_i
\end{bmatrix} > 0, \quad \forall (i, j) \in [1, d] \times [1, d], \quad (11)$$

then NCS (1) illustrated in Figure 2 is stabilisable via the output feedback control law

$$u(t) = Y_{l(k-1)}Z_{l(k-1)}^{-1}\tilde{y}(t), \quad t \in [t_k, t_{k+1}), \quad \forall k \in \mathbb{N} \quad (12)$$

Proof: We first prove that there exists a feedback gain set $\{F_1, F_2, \ldots, F_d\}$ guaranteeing the uniformly asymptotical stability of switched system (4) for arbitrary switching.

It is easy to show that the switched system (4) can be represented equivalently by

$$z(k + 1) = \sum_{j=1}^d \sum_{i=1}^d \xi_{ij}(k)A_{ij}z(k), \quad k \in \mathbb{N}, \quad (13)$$

where $A_{ij} = A_i^t + \sum_{l=0}^{L-1} A_i^t B_l F_l C$, and $\xi_{ij}(k)$ takes its values as 0 and 1. Based on (13), we know that every number pair $(i, j) \in [1, d] \times [1, d]$ denotes only one subsystem, and $\xi_{ij}(k) = 1$ means that the system matrix is $A_{ij}$ if the number pair $(i, j)$ subsystem is active and 0 otherwise. For switched system (13), we adopt the following switched Lyapunov function:

$$V(k, z(k)) = z^T(k) \sum_{i=1}^d \xi_i(k)P_i z(k), \quad (14)$$

where $P_i$ is the parameter to be designed.
Now, we will prove that the switched system (13) is uniformly asymptotically stable under the conditions (10) and (11).

In fact, the difference of (14) along the trajectory of switched system (13) is

\[
\Delta V(z(k)) = z^T(k + 1) \sum_{i=1}^{d} \xi_i(k + 1) P_i z(k + 1) - z^T(k) \sum_{i=1}^{d} \xi_i P_i z(k)
\]
\[
= z^T(k) \sum_{i=1}^{d} \sum_{j=1}^{d} \xi_i(k) P_i z(k)
\]
\[
 \times \left( A^i + \sum_{l=0}^{i-1} A^i B F_j C \right) \left( A^i + \sum_{l=0}^{i-1} A^i B F_j C \right)^T P_i (A^i + \sum_{l=0}^{i-1} A^i B F_j C) \]}

Then, according to the Lyapunov stability theory, switched system (13) is uniformly asymptotically stable if

\[
\left( A^i + \sum_{l=0}^{i-1} A^i B F_j C \right)^T P_i \left( A^i + \sum_{l=0}^{i-1} A^i B F_j C \right) - P_j < 0.
\]

From Lemma 2.3, we know that the inequality above holds if and only if

\[
\begin{bmatrix}
   P_j & \left( A^i + \sum_{l=0}^{i-1} A^i B F_j C \right)^T P_i \\
   P_i \left( A^i + \sum_{l=0}^{i-1} A^i B F_j C \right) & P_i
\end{bmatrix} > 0.
\]

(15)

By post- and pre-multiplying both sides of inequality (15) using the matrix \( \text{diag}(P_i^{-1}, P_i^{-1}) \) and letting \( X_i = P_i^{-1}, \ C X_i = Z_i C \) and \( Y_i = F_i Z_i \), we can get (11).

Next, we prove that NCS (1) is stabilisable under conditions (10) and (11).

In fact, based on the analysis in Section 2, when we apply Schedule Algorithm 1 to NCS (1), we can get the state evolution equation of the close-loop system as follows:

\[
x(t+1) = Ax(t) + B F_{r(k-1)} C x(t_k), \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}.
\]

For (16), we adopt the following packet-loss-dependent Lyapunov function:

\[
V(t) = x^T(t) P_{r(k-1)} x(t), \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}.
\]

The difference of (14) along the trajectory of NCS (16) is

\[
\Delta V(x(t_k)) = V(x(t_{k+1})) - V(x(t_k))
\]
\[
= z^T(k + 1) \sum_{i=1}^{d} \xi_i(k + 1) P_i z(k + 1)
\]
\[
- z^T(t) \sum_{i=1}^{d} \xi_i(t) P_i z(k)
\]
\[
= \Delta V(z(k)).
\]

Since switched system (4) is uniformly asymptotically stable proved by Lyapunov function (14), we know that \( \Delta V(z(k)) < 0 \), that is, \( \Delta V(x(t_k)) < 0 \). From Lemma 3.2, we obtain that NCS (1) is stabilisable under given conditions, and (27) is a stabilising controller. Thus, this completes the proof.

Similarly, for the case that packet losses occur both in the forward and backward channels, we have the following theorem for NCS (1) illustrated in Figure 1 and its proof can be derived by referring to that of Theorem 3.3.

**Theorem 3.4:** If there exist \( d \) symmetric positive definite matrices \( P_i \) and matrices \( F_i \) with \( i \in [1, d] \) such that

\[
\begin{bmatrix}
   P_j & \left( A^i + A^{i-1} B F_h C \right)^T + \sum_{l=0}^{i-2} A^i B F_j C \\
   \left( A^i + A^{i-1} B F_h C \right) + \sum_{l=0}^{i-2} A^i B F_j C & P_i^{-1}
\end{bmatrix} > 0,
\]

\( \forall (i, j, h) \in [1, d] \times [1, d] \times [1, d] \),

then NCS (1) illustrated in Figure 1 is stabilisable via the output feedback control law

\[
u(t) = F_{r(k-1)} \tilde{y}(t), \quad t \in [t_k + 1, t_{k+1} + 1), \quad \forall k \in \mathbb{N}.
\]

**Remark 3:** From the proof of Theorem 3.3, we know that (14) has been taken as a switched Lyapunov function for the switched system (13). In fact, (14) implies a packet-loss-dependent Lyapunov function for NCS (1). Let \( r(t_k) = t_{k+1} - t_k = i \), then (17) is exactly (14).

Notice that inequalities in (18) are non-convex due to containing \( P_i^{-1} \), which can be solved by the cone complementarity linearisation method (Ho and Lu 2003). The conditions provided in Theorem 3.3
contain an equation, we cannot solve them by the LMI toolbox directly. In order to obtain conditions which are easy to solve, we assume that \( \text{rank}(C) = p \). For any \( i \in [1, d] \), we also assume that \( X_i \) satisfies \( X_i = \mathcal{N}(X_0, 0, 0, X_N)^T \), where \( X_i \in \mathbb{R}^{p \times p} \), \( X_2 \in \mathbb{R}^{(n-p) \times (n-p)} \), and based on Lemma 2.4, we can get another sufficient condition for output stabilisation in the form of LMIs as follows.

**Corollary 3.5:** If there exist symmetric positive definite matrices \( X_i = \mathcal{N}(X_0, 0, 0, X_N)^T \) and matrices \( Y_i \) with \( i \in [1, d] \) such that (11) holds, then NCS (1) illustrated in Figure 2 is stabilisable via the output feedback controller

\[
u(t) = C_0 X_1^{-1} C_0^{-1} M^T.
\]

where \( Z_i = MC_0 X_1^{-1} C_0^{-1} M^T \).

**4. Stabilisation of NCSs with Markovian packet-loss process**

In this section, we suppose that the packet loss of NCSs is a Markovian process. Sufficient conditions for the stabilisation via output feedback are derived, and the stabilising output feedback controllers are designed by resolving some matrix inequalities.

Here, we still use Schedule Algorithm 1 to stabilise NCS (1), and the states of NCS (1) with Markovian packet-loss process at the update instants can be described as follows:

\[
z(k + 1) = \tilde{A}_{\eta(k)} z(k),
\]

where \( \tilde{A}_{\eta(k)} \in \mathcal{O} \), and for all \( k > 1 \), \( \eta(k) = (r(k), r(k-1)) \in [1, d] \times [1, d] \), \( \eta(1) = (r(1), 1) \), and \( r(k) \) is a Markovian chain as defined in Definition 2.2.

**Definition 4.1:** NCS (1) with the Markovian packet-loss process defined in Definition 2.2 is said to be mean square (MS) stable if \( \lim_{t \to \infty} E[\|x(t)\|^2] = 0 \) for any initial state \( x_0 \).

**Lemma 4.2:** NCS (1) illustrated in Figure 2 with the Markovian packet-loss process defined in Definition 2.2 is MS stabilisable if there exist positive definite matrices \( P_i, i \in [1, d] \), such that

\[
\Upsilon_j = \sum_{i=1}^{d} p_j \left( A^i + \sum_{l=0}^{i-1} A^l B F_j C \right)^T P_i
\]

\[
\times \left( A^i + \sum_{l=0}^{i-1} A^l B F_j C \right) - P_j < 0, \quad \forall j \in [1, d].
\]

**Proof:** We only need to prove that (20) is MS stable. Taking (17) as a Lyapunov function for (20), we have

\[
E[\Delta V(x(k))r(k-1) = j, r(k-2) = h]
\]

\[
= E[\Delta^T (k + 1) P_{r(k)} x(k + 1)]
\]

\[
\times r(k-1) = j, r(k-2) = h] - x^T(k) P_j x(k)
\]

\[
= x^T(k) \left( \sum_{i=1}^{d} p_j \left( A^i + \sum_{l=0}^{i-1} A^l B F_j C \right)^T \right)
\]

\[
\times P_i \left( A^i + \sum_{l=0}^{i-1} A^l B F_j C \right) - P_j \right) x(k)
\]

Thus, if \( \Upsilon_j < 0 \) for all \( j \in [1, d] \), we have \( \lim_{k \to \infty} E[V(k)] = 0 \). Since \( P_i, i \in [1, d] \) are positive definite matrices, we have \( \lim_{k \to \infty} E[\|x(k)\|^2] = 0 \). By Definition 4.1, we know that (20) is MS stable. Thus, this completes the proof.

Based on Lemmas 2.3 and 4.2, it is easy to get the following sufficient condition for the stabilisation via output feedback.

**Theorem 4.3:** NCS (1) illustrated in Figure 2 with the Markovian packet-loss process defined in Definition 2.2 is MS stabilisable if there exist symmetric positive definite matrices \( X_i \), and matrices \( W_i, G_i, Y_i \) with \( i \in [1, d] \) satisfying

\[
CG_i = W_i C, \quad (22)
\]

and

\[
[\begin{array}{cc}
\Lambda & Q_i^T \\
Q_i & X_i
\end{array}] > 0, \quad \forall i \in [1, d],
\]

where

\[
Q_i = [\sqrt{p_i}(AG_i + BY_i C) \cdots \sqrt{p_i}(A^d G_i + B_d Y_i C)],
\]

\[
\Lambda = \text{diag}(G_1 + G_1^T - X_1 \cdots G_d + G_d^T - X_d),
\]

\[
B_j = \sum_{l=0}^{i-1} A^l B_l.
\]

Moreover, \( u(t) = F_{r(k-1)} \bar{u}(t) = Y r(k-1) W^{-1}_{r(k-1)} \bar{u}(t), t \in [t_k, t_k+1) \) is a stabilising output feedback control law.

In Theorem 4.3, as in Oliveira, Bernhardsson, and Wittenmark (1999), we introduce new additional matrices \( \tilde{G}_i \) to the stabilising conditions in which the positive definite matrices \( X_i \) are not involved in any product with other matrices. This presents the extra degree of freedom. However, we can see that the conditions in Theorem 4.3 contain an equation like those in Theorem 3.3. Similarly, we assume that \( \text{rank}(C) = p \). By using Lemma 2.4, we can get another sufficient condition in the form of LMIs for the output
stabilisation of NCS (1) with the Markovian packet-loss process.

**Corollary 4.4:** If there exist symmetric positive definite matrices \( X_i \), symmetric matrices \( G_i = M [ G_{i1} 
abla G_{i2} ] N^T \) and matrices \( Y_i \) with \( i \in [1,d] \) such that

\[
\begin{bmatrix}
\Lambda & Q_i^T \\
Q_i & X_i
\end{bmatrix} > 0, \quad \forall i \in [1,d],
\]  

(24)

where

\[
Q_i = [\sqrt{p_{i1}}(A_{i1} + BY_i) C] \cdots \sqrt{p_{id}}(A_{id} + BY_i) C],
\]

\[
\Lambda = \text{diag}(2G_1 - X_1 \cdots 2G_d - X_d), \quad B_j = \sum_{l=0}^{i-1} A^l B,
\]

then NCS (1) illustrated in Figure 2 with the Markovian packet-loss process defined in Definition 2.2 is MS stabilisable via the output feedback controller

\[
u(t) = Y_{r(k-1)} \tilde{W}_{r(k-1)} \hat{x}(t), \quad t \in [t_k, t_{k+1}),
\]

where \( \tilde{W}_i = MC_0 G_i^{-1} C_0^{-1} N^T, \forall i \in [1, d] \), and \( C = M[ C_0 0 ] N^T \) is the singular value decomposition of \( C \).

**Remark 4:** For NCS (1) illustrated in Figure 2 with a Markovian packet-loss process, we can find the maximum allowable bound of packet loss by solving the following optimal problem:

\[
\max_{X_i, Y_i, G_{i1}, G_{i2}} d
\]

subject to (24).

Similarly, for an NCS with Markovian packet-loss processes occurring in both the backward and forward channels, which is illustrated in Figure 1, we have the following results.

**Theorem 4.5:** If there exist d symmetric positive definite matrices \( P_i \) and matrices \( F_i \) with \( i \in [1, d] \) such that

\[
\begin{bmatrix}
\Lambda & Q_i^T \\
Q_i & P_i
\end{bmatrix} > 0, \quad \forall (i,j) \in [1,d] \times [1,d],
\]  

(26)

where

\[
\Lambda = \text{diag}(P_1^{-1} \cdots P_d^{-1}),
\]

\[
Q_i = [\sqrt{p_{i1}}(A + BF_i) C] \cdots \sqrt{p_{id}}(A + BF_i) C],
\]

\[
\times \left( A^d + A^{d-1} BF_i C + \sum_{l=0}^{d-2} A^l BF_i C \right),
\]

then NCS (1) illustrated in Figure 1 is MS stabilisable via the output feedback control law

\[
u(t) = F_{r(k-1)} \hat{x}(t), \quad t \in [t_k + 1, t_{k+1} + 1), \quad \forall k \in \mathbb{N}.\]

(27)

5. **Stabilisation via dynamic output feedback**

In this section, we assume that the network exists only between the sensor and the controller as illustrated in Figure 3, and we study dynamic output feedback stabilisation of NCSs by the proposed design technique given above. The NCS with a dynamic output feedback controller are given by

\[
x(t + 1) = A x(t) + B u(t),
\]

\[
y(t) = C x(t),
\]

\[
\hat{x}(t + 1) = (A - LC) \hat{x}(t) + Bu(t) + L \tilde{y}(t),
\]

\[
u(t) = F(t) \hat{x}(t),
\]

(28)

where piecewise continuous gain matrix functions \( F(t) \) and \( L(t) \) are to be designed.

With two sets of candidate gains \( \{F_1, F_2, \ldots, F_d\} \) and \( \{L_1, \ldots, L_d\} \) to be designed, we propose the following feedback schedule algorithm to stabilise NCS (28).

**Schedule Algorithm 3:** For any update instant \( t_k \), assuming that there is a counter along with the controller, which records the length of the last transmission interval \( [t_{k-1}, t_k) \), we take the packet-loss-dependent feedback gains as \( F_{t_k-t_{k-1}}, L_{t_k-t_{k-1}} \) in the following transmission interval \( [t_k, t_{k+1}) \), i.e.

\[
u(t) = F_{t_k-t_{k-1}} \hat{x}(t), \quad \text{and} \quad \hat{x}(t + 1) = (A - L_{t_k-t_{k-1}} C) \hat{x}(t) + Bu(t) + L_{t_k-t_{k-1}} \hat{y}(t), \quad t \in [t_k, t_{k+1}).
\]

Applying Schedule Algorithm 3 to stabilise the NCS by letting \( r(k) = t_{k+1} - t_k, \eta(k) = (r(k), r(k-1)) \), we can get the states of NCS (28) at the update steps described as follows:

\[
z(k + 1) = \hat{A}_{r(k)} z(k),
\]

(29)

![](image.png)

Figure 3. Structure of an NCS with dynamic output feedback controller.
where $z(k) = [x^T(t_k) e^T(t_k)]^T$, $e(t_k) = x(t_k) - \hat{x}(t_k)$, and $A_{n(k)} = A_{r(k-1)}^{-1} \Gamma_{r(k-1)} + \sum_{h=0}^{n(k)-2} A_{r(k-h)} \Upsilon_{r(k-h)}$, with

$$
\Lambda_{r(k-1)} = \begin{bmatrix} A + BF_{r(k-1)} & -BF_{r(k-1)} \\ L_{r(k-1)}C & A - L_{r(k-1)}C \end{bmatrix},
$$

$$
\Gamma_{r(k-1)} = \begin{bmatrix} A + BF_{r(k-1)} & -BF_{r(k-1)} \\ 0 & A - L_{r(k-1)}C \end{bmatrix},
$$

$$
\Upsilon_{r(k-1)} = \begin{bmatrix} 0 & 0 \\ -L_{r(k-1)}C & 0 \end{bmatrix}.
$$

Based on the analysis above, in order to get the stabilising conditions for (28) with the arbitrary packet-loss process, we only need to discuss the stabilisation of the switched system (29).

For (29), taking the switched Lyapunov function

$$
V(k, z(k)) = z^T(k) P_{r(k)} z(k),
$$

where $P_{r(k)} \in \mathbb{R}^{n \times n}$ is positive definite, we have the following results, and we omit its proof to avoid repetition.

**Theorem 5.1:** If there exist symmetric positive definite matrices $X_i = N_i^T X_i N_i^T$ and matrices $Y_i$, $Z_i$ with $i \in [1, d]$, such that

$$
\begin{bmatrix}
\tilde{X}_j \\
\tilde{A}_j^T
\end{bmatrix} > 0, \quad \forall (i, j) \in [1, d] \times [1, d],
$$

where for all $(i, j) \in [1, d] \times [1, d]$, $\tilde{A}_j = \Lambda_j^{-1} \Gamma_j + \sum_{h=0}^{i-2} \Lambda_h \Upsilon_h$, and

$$
\Lambda_j = \begin{bmatrix} AX_j + BY_j & -BY_j \\ Z_j C & AX_j - Z_j C \end{bmatrix},
$$

$$
\Gamma_j = \begin{bmatrix} AX_j + BY_j & -BY_j \\ 0 & AX_j - Y_j C \end{bmatrix},
$$

$$
\tilde{X}_j = \begin{bmatrix} X_i \\
0
\end{bmatrix},
$$

$$
\tilde{Y}_j = \begin{bmatrix} -Z_j C \\
0
\end{bmatrix},
$$

then NCS (28) is stabilisable via the dynamic output feedback law

$$
u(t) = Y_{r(k-1)} G_{r(k-1)}^{-1} \tilde{x}(t), \quad t \in [t_k, t_{k+1}),
$$

$$\tilde{x}(t+1) = (A - Z_{r(k-1)} \tilde{X}_{r(k-1)} C) \tilde{x}(t) + Bu(t) + Z_{r(k-1)} \tilde{Y}_{r(k-1)} \tilde{y}(t), \quad t \in [t_k, t_{k+1}),
$$

where $\tilde{X}_i = MC_0 X_i^{-1} C_0^{-1} M^T$, and $C = M [C_0 \ 0]^T$ is the singular value decomposition of $C$.

**Theorem 5.2:** If there exist symmetric positive definite matrices $X_i$, symmetric matrices $G_i = N_i^T G_i N_i^T$ and matrices $Y_i$ with $i \in [1, d]$ such that

$$
\begin{bmatrix}
\Lambda \\
\tilde{Q}_i \tilde{X}_i
\end{bmatrix} > 0, \quad \forall i \in [1, d],
$$

where

$$
\Lambda = \text{diag}(2 \tilde{G}_1 - \tilde{X}_1 \cdots 2 \tilde{G}_d - \tilde{X}_d),
$$

$$
\tilde{Q}_i = [\sqrt{p_i} \tilde{A}_i: \cdots: \sqrt{p_d} \tilde{A}_d],
$$

and for all $(i, j) \in [1, d] \times [1, d]$, $\tilde{A}_j = \tilde{A}_j^{-1} \Gamma_j + \sum_{h=0}^{i-2} \tilde{A}_h \Upsilon_h$, with

$$
\tilde{A}_j = \begin{bmatrix} AG_j + BY_j & -BY_j \\ Z_j C & AG_j - Z_j C \end{bmatrix},
$$

$$
\tilde{\Gamma}_j = \begin{bmatrix} AG_j + BY_j & -BY_j \\ 0 & AG_j - Y_j C \end{bmatrix},
$$

$$
\tilde{\Upsilon}_j = \begin{bmatrix} 0 \\
-Z_j C \\
0
\end{bmatrix},
$$

then NCS (28) is stabilisable via the dynamic output feedback law

$$
u(t) = N_{r(k-1)} G_{r(k-1)}^{-1} \tilde{x}(t), \quad t \in [t_k, t_{k+1}),
$$

$$\tilde{x}(t+1) = (A - Z_{r(k-1)} \tilde{X}_{r(k-1)} C) \tilde{x}(t) + Bu(t) + Z_{r(k-1)} \tilde{Y}_{r(k-1)} \tilde{y}(t), \quad t \in [t_k, t_{k+1}),
$$

where $\tilde{X}_i = MC_0 X_i^{-1} C_0^{-1} M^T$, and $C = M [C_0 \ 0]^T$, is the singular value decomposition of $C$.

6. Numerical examples

In this section, some numerical examples and simulations are worked out to demonstrate the effectiveness of the proposed design techniques.

**Example 6.1:** Consider a second-order NCS

$$
x(t+1) = \begin{bmatrix} 1.166 & 0.209 \\
-0.123 & -0.889
\end{bmatrix} x(t) + \begin{bmatrix} 1 \\
1
\end{bmatrix} u(t),
$$

$$
y(t) = \begin{bmatrix} 2 & 1
\end{bmatrix} x(t),
$$

$$
u(t) = F_i \tilde{y}(t), \quad \forall i \in [1, d],
$$

where the output feedback gains $F_i, i \in [1, d]$ are to be designed. Notice that the open-loop system is unstable since one of its poles (1.1534) is outside the unit disk. Here, we suppose that the maximum transmission interval $d = 3$ which means that the 66% of the packets can be lost, and take the initial state as $x_0 = [-10 \ 10]^T$. When the packet loss of the NCS (34) is arbitrary,
solving LMI (11) by using the Matlab LMI Toolbox, we obtain the following output feedback gains:

\[ F_1 = 0.2457, \quad F_2 = 0.2159, \quad F_3 = 0.1974. \]

When the distribution of transmission interval is 1, 2, 3, 1, 2, 3, ..., the step response of NCS (34) is shown in Figure 4. It is clear from the figure that even in such a case, the system can still be effectively stabilised via the switched output feedback given above. Figure 5 depicts the trajectory of the system when no packet loss occurs.

In addition, we suppose the packet loss of NCS (34) is a Markovian process whose transition probability matrix is

\[
P = \begin{bmatrix}
0.1 & 0.2 & 0.2 \\
0.3 & 0.1 & 0.3 \\
0.6 & 0.7 & 0.5
\end{bmatrix}.
\]

We solve the LMIs in Corollary 4.4 by using the Matlab LMI Toolbox, and obtain the feedback gains:

\[ F_1 = 0.2213, \quad F_2 = 0.2254, \quad F_3 = 0.2300. \]

When the initial state is \( x_0 = [-10 \ 10]^T \), the step response is shown in Figure 6 under the output feedback above. The small circles in Figure 7 simulate the number of packet loss at the time instants when the packet loss occurs. We can see clearly from the figures that the system can still be effectively stabilised via the switched output feedback given above.

**Example 6.2:** Consider a third-order NCS with multiple inputs and multiple outputs

\[
x(t + 1) = \begin{bmatrix}
0.5055 & 0 & -0.3158 \\
0.2245 & 0.6758 & -0.0432 \\
0.1 & 0 & 1.4553
\end{bmatrix} \begin{bmatrix}
x(t) \\
0 \\
0
\end{bmatrix}.
\]
\[ y(t) = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ 0 & 0.1 & 0.5 \end{bmatrix} x(t), \]
\[ u(t) = F_i \tilde{y}(t), \quad \forall i \in [1, d], \quad (36) \]

where the output feedback gains \( F_i, i \in [1, d] \) are to be designed. Notice that the open-loop system is unstable since one of its poles (1.4208) is outside the unit disk.

Next, we design packet-loss-dependent controllers to stabilise the unstable third-order system with bounded packet loss. Suppose that the maximum transmission interval \( d = 3 \) which means that the 66% of the packets can be lost, and the initial state is \( x_0 = \begin{bmatrix} -2 & 2 & 5 \end{bmatrix}^T \).

When the packet loss of the NCS (36) is arbitrary, solving LMI (11) by using the Matlab LMI Toolbox, we obtain the following feedback gains:

\[
\begin{align*}
F_1 &= \begin{bmatrix} 0.4750 & -1.3837 \\ -1.6840 & -0.2329 \end{bmatrix}, \\
F_2 &= \begin{bmatrix} 0.5077 & -1.2979 \\ -1.3433 & -0.2367 \end{bmatrix}, \\
F_3 &= \begin{bmatrix} 0.4834 & -1.1404 \\ -1.2155 & -0.1794 \end{bmatrix}.
\end{align*}
\]

When the distribution of transmission interval is 1, 2, 3, 1, 2, 3, ..., the step response of NCS (36) is shown in Figure 8. It is clear from the figure that even in such case, the system can still be effectively stabilised via the switched output feedback given above. Figure 9 depicts the step response of the system when no packet loss occurs.

In addition, when the packet loss of NCS (36) is a Markovian process whose transition probability matrix is (35) and the initial state is \( x_0 = \begin{bmatrix} 8 & 10 & 2 \end{bmatrix}^T \). Solving the LMI in Corollary 4.4 by using the Matlab LMI Toolbox, we obtain the following output feedback gains:

\[
\begin{align*}
F_1 &= \begin{bmatrix} 0.9574 & -1.4302 \\ -1.5181 & 0.0195 \end{bmatrix}, \\
F_2 &= \begin{bmatrix} 0.9466 & -1.3724 \\ -1.6770 & 0.0733 \end{bmatrix}, \\
F_3 &= \begin{bmatrix} 0.3234 & -0.8655 \\ -1.9608 & 0.0754 \end{bmatrix}.
\end{align*}
\]

When the initial state is given by \( x_0 = \begin{bmatrix} -8 & 10 & 2 \end{bmatrix}^T \), the step response is shown in Figure 10 under the output feedback above. The small circles in Figure 11 simulate the number of packet loss at the time instants when the packet loss occurs. From the figures, we can see clearly...
that the system can still be effectively stabilised via the switched output feedback given above.

Next, we will show the advantage of our design over that of Yu et al. (2004). We only consider the case that the packet loss of NCS (34) is arbitrary process. Solving the LMI (27) presented in Theorem 3 in Yu et al. (2004), we get the feedback gain:

$$F = \begin{bmatrix} 0.5176 & -1.1010 \\ -2.8463 & 0.1480 \end{bmatrix}$$.

When the initial state is $x_0 = [2 \ 1 \ -1]^T$, Figure 12 depicts the trajectory of the system when no packet loss occurs. It is clear from the figure that the system can be effectively stabilised via the output feedback given above. However, when the distribution of transmission interval is 1, 2, 3, 1, 2, 3, ..., the step response of NCS (36) is shown in Figure 13 where the state trajectory is not convergent. Thus, by the design provided in Yu et al. (2004), the system cannot be stabilised via the single output feedback when the distribution of transmission interval is 1, 2, 3, 1, 2, 3, ....

7. Conclusions

This article has discussed the stabilisation problem for NCSs with bounded packet loss. We have modelled such NCSs as switched systems, and derived sufficient conditions for output feedback stabilisation of NCSs by using a packet-loss-dependent Lyapunov function. Moreover, stabilising packet-loss-dependent output feedback controllers have been designed for two types of packet-loss processes: one is the arbitrary packet-loss process, and the other is the Markovian packet-loss process. Furthermore, by a similar design technique, we also have studied the stabilisation via dynamic output feedback, and established the stabilising conditions for the NCSs. Two numerical examples and several simulations have been given to demonstrate the effectiveness of the proposed design techniques.

References


