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Systematic design and realization of double-negative acoustic metamaterials by topology optimization

Hao-Wen Dong a, b, *, Sheng-Dong Zhao c, 1, Peijun Wei a, Li Cheng b, Yue-Sheng Wang d, e, *, Chuanzeng Zhang f

a Department of Applied Mechanics, University of Science and Technology Beijing, Beijing, 100083, PR China
b Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong, PR China
c School of Mathematics and Statistics, Qingdao University, Qingdao, 266071, PR China
d Department of Mechanics, Beijing Jiaotong University, Beijing, 100044, PR China
e Department of Mechanics, School of Mechanical Engineering, Tianjin University, Tianjin, 300350, PR China
f Department of Civil Engineering, University of Siegen, D-57068, Siegen, Germany

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A B S T R A C T
Double-negative acoustic metamaterials (AMMs) offer the promising ability of superlensing for applications in ultrasonography, biomedical sensing and nondestructive evaluation. However, the systematic design and realization of broadband double-negative AMMs are still missing, which hinder their practical implementations. In this paper, under the simultaneous increasing or non-increasing mechanisms, we develop a unified topology optimization framework involving different microstructure symmetries, minimal structural feature sizes and dispersion extents of effective parameters. The optimization framework is applied to conceive the heuristic resonance-cavity-based and space-coiling metamaterials with broadband double negativity. Meanwhile, we demonstrate the essences of double negativity derived from the novel artificial multipolar LC (inductor-capacitor circuit) and Mie resonances which can be induced by controlling mechanisms in optimization. Furthermore, abundant numerical simulations validate the corresponding double negativity, negative refraction, enhancement of evanescent waves and subwavelength imaging. Finally, we experimentally show the desired broadband subwavelength imaging by using the 3D-printed optimized space-coiling metamaterial. The present design methodology provides an ideal approach for constructing the constituent “atoms” of metamaterials according to any artificial physical and structural requirements. In addition, the optimized broadband AMMs and superlens lay the structural foundations of subwavelength imaging technology.

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1. Introduction

Due to the fantastic wave characteristics, metamaterials [1–6] designed by engineering the subwavelength microstructures offer novel and exceptional opportunities for manipulating and controlling wave propagation, revealing the broad application prospects in various fields such as mechanics, materials, optics, electromagnetism, acoustics, and thermotics, etc. In general, conventional materials drive their wave motions from the properties of intrinsic atoms or molecules; metamaterials provide novel possibilities for constructing artificial “meta-atoms” (microstructures) with special geometry, physical features and spatial arrangements, thus bringing out many new functionalities. Electromagnetic metamaterials and metadevices can achieve a wide range of exotic electromagnetic responses, including negative refractive index, zero refractive index, optical chirality, anisotropy and hyperbolicity. Inspired by the optical metamaterials, acoustic metamaterials (AMMs) [2,7], elastic metamaterials [8–12], mechanical metamaterials [5] and even graphene metamaterials [13] have been developed in many ways. Like other types of metamaterials, creating suitable building blocks of microstructures is the most fundamental and pivotal point for AMMs which exhibit diverse combinations of effective constitutive parameters—the mass density $\rho_{\text{eff}}$ and bulk modulus $K_{\text{eff}}$. In the quadrants of AMMs, reported representative cases are single negativity ($\rho_{\text{eff}}<0$, $K_{\text{eff}}>0$, $\rho_{\text{eff}}>0$, $K_{\text{eff}}<0$), double negativity ($\rho_{\text{eff}}<0$, $K_{\text{eff}}<0$), single positive negativity ($\rho_{\text{eff}}>0$, $K_{\text{eff}}<0$), and single double negativity ($\rho_{\text{eff}}<0$, $K_{\text{eff}}<0$, $\rho_{\text{eff}}>0$, $K_{\text{eff}}<0$) conditions. However, the systematic design and realization of broadband double-negative AMMs are still missing, which hinder their practical implementations. In this paper, under the simultaneous increasing or non-increasing mechanisms, we develop a unified topology optimization framework involving different microstructure symmetries, minimal structural feature sizes and dispersion extents of effective parameters. The optimization framework is applied to conceive the heuristic resonance-cavity-based and space-coiling metamaterials with broadband double negativity. Meanwhile, we demonstrate the essences of double negativity derived from the novel artificial multipolar LC (inductor-capacitor circuit) and Mie resonances which can be induced by controlling mechanisms in optimization. Furthermore, abundant numerical simulations validate the corresponding double negativity, negative refraction, enhancement of evanescent waves and subwavelength imaging. Finally, we experimentally show the desired broadband subwavelength imaging by using the 3D-printed optimized space-coiling metamaterial. The present design methodology provides an ideal approach for constructing the constituent “atoms” of metamaterials according to any artificial physical and structural requirements. In addition, the optimized broadband AMMs and superlens lay the structural foundations of subwavelength imaging technology.
$K_{\text{eff}} > 0; \rho_{\text{eff}} > 0, K_{\text{eff}} < 0)$, double negativity ($\rho_{\text{eff}} < 0, K_{\text{eff}} < 0$), double positivity ($\rho_{\text{eff}} > 0, K_{\text{eff}} > 0$) near-zero mass density ($\rho_{\text{eff}} \approx 0$), and even double-zero index ($\rho_{\text{eff}} = 0, 1/K_{\text{eff}} = 0$). Benefitting from the exotic effective properties, AMMs offer great potential for applications in low-frequency isolation, sound field modulating, energy harvesting, perfect absorption, negative refraction, cloaking and nonreciprocal acoustic devices, and thus attracting the widespread and continuous attention during the past two decades. In acoustics, one of the most promising functionalities of AMMs is the subwavelength superlensing, warranting high-resolution ultrasonic imaging for medicine and industry [14,15]. Although the anamorphic effective refractive index [16] or phase difference [17] can enable the gradient metamaterials to focus waves in a focal plane, the conspicuous shortcoming is that their imaging resolutions cannot essentially break the diffraction limit. Alternatively, several strategies using microstructures [18–21] with different features of effective parameters can collect and exploit the evanescent wave field for the subwavelength details. One prominent technique for subwavelength imaging is the double-negative superlenses [20]. It can cause the negative refraction, and then bring the diverging waves to reconvene and amplify the evanescent waves in the near field. Another approach relies on the anisotropic metamaterials [18,19] which can convert the coupling of near-field-emitted by subwavelength objects into propagating waves. In addition to the above strategies, time-reversal technique can also control and focus the subwavelength waves. By virtue of the Helmholtz resonators, the temporal response is recorded, flipped in time and radiated back, achieving the subwavelength focusing [21]. Furthermore, recent research indicates that, for metamaterials having relatively high index within a slow medium, the excitation of guided acoustic modes can transmit only subwavelength information to generate the subwavelength edge-based imaging [22]. In consideration of the operating bandwidth, in particular, the double-negative and hyperbolic metamaterials are more suitable for the broadband acoustic subwavelength focusing and imaging applications. In most cases, the hyperbolic metamaterials are highly desirable for the structure design. Only the three-dimensional membrane-type metamaterials [23] or layers of perforated plates [24] can produce the extremely anisotropic dispersion relations at the subwavelength scale. Nevertheless, double-negative metamaterials allow three choices including the coupled filter-element [25], coupled-membrane [26] and space-coiling structures [7]. Therefore, it is necessary to go into constructing the double negativity and the corresponding subwavelength superlensing.

In principle, the acoustic double negativity implies the several building blocks or specific elements supporting multiple overlapping resonances. The first approach is combining two resonating structures, membranes and Helmholtz resonators, to guarantee that their symmetric eigenmodes occur in the same dispersive frequency range, such as the example given in Ref. [25] to obtain the double negativity from 240 to 450 Hz based on a periodic array of interspaced membranes and side holes. Alternatively, coupled-membrane resonators can also lead to double negativity with monopolar and dipolar eigenmodes in the range of 520–830 Hz [26]. In addition, the combination of clamped membranes and side branches can generate both anisotropic and double-negative properties as well [27]. It is found that detuned Helmholtz resonators with optimized coupling can also give rise to double-negative bands [28]. Moreover, the double negativity can be obtained through cavities drilled in the waveguide which contains the Helmholtz resonators for negative bulk modulus and a structured shell for negative mass density [29]. Another strategy, based on the ultra-slow Mie resonators consisting of macroporous microbeads [30], can also generate the negative acoustic index. Moreover, by coiling up space, the labyrinthine structures composed of hard solid plates inserted into the background fluid can cause the large phase delay and form the band folding in the low frequency range [7], thus exhibiting a frequency dispersive spectrum of a large refractive index not found in nature and double negativity without the traditional resonant elements. While the aforementioned methods preliminarily realized double negativity, the following problems and challenges need to be solved to achieve broadband double negativity for the subwavelength imaging. Firstly, from the perspective of microstructure design, the primary double-negative AMMs mainly depend on membranes or space-coiling structures. Compared with the membrane-type metamaterials, the space-coiling structures have attracted more attention for its strong control over the effective parameters and easy implementation. Because of the difficulties in constructing the pertinent zigzag path, however, most research only focus on the flexible phase manipulation [31] instead of the broadband double negativity within the spectrum of interest. Meanwhile, apart from the space-coiling metamaterials, designing various types of double-negative solid-air AMMs becomes a pressing issue to offer more choices for practical imaging applications. Secondly, regarding double-negative mechanism, existing AMMs mainly utilize the LC (inductor-capacitor circuit) resonance [25] induced from different resonant elements for the Mie resonance [30] produced by the particles with high refractive index relative to the background medium. There are, however, few works about the LC-resonance double negativity based on the artificial structures without membranes. Mie-resonance double negativity in a broadband range also challenges the microstructure design. Thirdly, with respect to the double-negative bandwidth, resonance-based AMMs usually suffer from frequency dispersions and narrow bands. Hence, it is imperative to broaden the frequency range of double-negative AMMs for solidifying their roles in diverse applications [32]. But in general, the above three issues are collectively limited by the manual and empirical design strategy.

With the advent of the burgeoning 3D printing technology, topology optimization has successively applied to the design of metamaterials to achieve the desired performance [11,12,33–38]. Since AMMs have shown the unprecedented functionalities on wave manipulations, several topology optimization studies of AMMs were subsequently reported in recent years [35,39,40]. However, these works mainly focused on the optimization of propagation responses [35] or positive wave parameters [39]. Furthermore, the topology-optimized AMMs only realized the expected narrow-band single negativity [40], anisotropic dispersion relation [35] and negative refraction [35], but lacking of the demonstration for subwavelength imaging. So topology optimization of AMMs is still in its infancy stage, although the increasing demand for more newfangled phenomena and acoustic devices is clearly foreseeable. Up to now, the inverse design of double-negative AMMs for airborne sound is still lacking, let alone the systematic optimization study. Moreover, unlike elastic metamaterials [8–12,41], the negative effective parameters not only depend on the LC resonance, but also are possibly dominated by the Mie resonance. Because of the uncertain mechanism for negativity, topology optimization of double-negative AMMs is full of challenges.

In this paper, to provide a comprehensive guidance on engineering the double-negative AMMs, we show for the first time that the broadband double-negative AMMs can be systematically designed through topology optimization. A unified topology optimization framework is constructed for obtaining the broadband double negativity within the prescribed low frequency range. The proposed framework considers several typical structural and physical characteristics of the microstructure, including unit-cell’s symmetry (i.e., the square, chiral and orthogonal symmetries),
minimal geometrical size (i.e., minimal size of the solid parts and the width of the air channels), variation trend and frequency dispersion of effective parameters. Band structures and retrieval of effective parameters demonstrate that the topology-optimized microstructures really exhibit the broadband double negativity. All metamaterials present here uncover two kinds of topological characteristics, i.e., the resonance-cavity-based and space-coiling layouts. Eigenstate analyses reveal that the optimization methodology under simultaneous increasing tendencies of the effective mass density and bulk modulus can give rise to the LC-resonance double negativity; whereas the simultaneous non-increasing tendencies can evoke the Mie-resonance double negativity. Then, the subwavelength negative refraction and acoustic imaging are numerically demonstrated after validating the equi-frequency surfaces and enhanced transmission of the evanescent waves. Finally, we fabricate a topology-optimized space-coiling AMM through 3D printing to successfully perform the subwavelength imaging in an acoustic experiment.

2. Topology optimization methodology

Consider the square-latticed microstructures consisting of solid and air elements for the acoustic wave propagation, as shown in Fig. 1. Based on the simulation model depicted in Fig. 1(a), the transmission and reflection coefficients of one microstructure can be calculated for the acoustic effective parameters retrieval, as long as the microstructure is symmetric along the direction of wave propagation. The dispersion relations of the microstructures can be characterized by the Floquet-Bloch theory. In topology optimization, the microstructure is divided into $N \times N$ pixels, in which air and solid are denoted by “0” and “1”, respectively. It is assumed that the microstructure has three representative types of symmetries during topology optimization, i.e., square, chiral and orthogonal symmetries, as illustrated in Fig. 1(c). Because of these symmetry assumptions, the design domain in the topology optimization changes from the whole unit-cell to a reduced space, see Fig. 1(c). Similarly, their corresponding irreducible Brillouin zones are displayed in Fig. 1(c).

2.1. Characterization of AMMs

To formulate the wave equations for the present high contrast (solid/air) wave problems shown in Fig. 1, the acoustic-structural interactions are ignored for simplicity. Because the solid can be regarded as perfectly hard, namely, the wave propagation is principally predominant in the background air [35,42]; it is a good approximation to take the solid as a fluid with very high stiffness and specific mass [42]. Therefore, we only consider the traditional acoustic wave equation:

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**Fig. 1. Schematic illustration of the square-latticed AMMs and representative microstructural symmetries.**

(a) Wave propagation model for calculating the transmission and reflection coefficients of one microstructure. The simulation domain is terminated with the infinite elements at the left and right boundaries. The top and bottom edges are set as rigid walls. Four probes are introduced in the model for the scattering coefficients retrieval. (b) Double-negative AMMs with desired wave manipulation. (c) Three representative symmetries (left: square symmetry; middle: chiral symmetry; right: orthogonal symmetry) investigated in topology optimization. The dashed and solid lines show the corresponding reduced design domains and edges of the first reduced Brillouin zone, respectively.
\[ \nabla \cdot \left( \rho^{-1}(r) \nabla p(r) \right) + \omega^2 \lambda^{-1}(r) p(r) = 0. \]  

(1)

where \( p \) is the acoustic pressure; and \( \lambda \) for acoustic case equals to the bulk modulus \( K \). According to the Floquet-Bloch theory, the acoustic pressure can be written as \( p(r) = e^{ikr}p_0(r) \), where \( k \) and \( p_0(r) \) are the Bloch wave vector of the first Brillouin zone and periodic function of \( r \), respectively. Considering the wave equation and boundary conditions, we can calculate the dispersion relation.

In our simulation, the complex sound pressures at the four probes \( x_1, x_2, x_3 \) and \( x_4 \) are respectively expressed as

\[ P_1 = (Ae^{ikx_1} + Be^{-ikx_1}) e^{-i\omega t}, \]
\[ P_2 = (Ae^{ikx_2} + Be^{-ikx_2}) e^{-i\omega t}, \]
\[ P_3 = (C e^{ikx_3} + D e^{-ikx_3}) e^{-i\omega t}, \]
\[ P_4 = (C e^{ikx_4} + D e^{-ikx_4}) e^{-i\omega t}. \]

(2)

where \( A, B, C \) and \( D \) denote the complex amplitudes of the positive- and negative-going plane waves; \( k_0 \) is the wave number in the background medium (air); \( \omega \) is the circular frequency; \( e^{ikx} \) represents the common time-harmonic factor which is omitted throughout the paper for the sake of brevity; \( x_1, x_2, x_3 \) and \( x_4 \) denote the corresponding distances of four probes relative to respective reference planes in the two ports. Since the pressures \( P_1, P_2, P_3 \) and \( P_4 \) can be directly obtained by the numerical simulation, the four coefficients \( A, B, C \) and \( D \) can be derived from Eq. (2) as

\[ A = -\frac{P_1 e^{-ikx_2} - P_2 e^{-ikx_1}}{2 \sin(k_0 x_1 - k_0 x_2)}, \]
\[ B = -\frac{i P_2 e^{-ikx_2} - P_1 e^{-ikx_1}}{2 \sin(k_0 x_1 - k_0 x_2)}, \]
\[ C = i \frac{P_3 e^{-ikx_3} - P_4 e^{-ikx_4}}{2 \sin(k_0 x_4 - k_0 x_3)}, \]
\[ D = i \frac{P_4 e^{-ikx_4} - P_3 e^{-ikx_3}}{2 \sin(k_0 x_4 - k_0 x_3)}. \]

(3)

The second order matrix relating the acoustic particle velocity and sound pressure on two faces of the microstructure in the simulation model shown in Fig. 1(a) is defined as the transfer matrix which is denoted as \( T \) with the elements \( T_{ij} \) \((i,j = 1, 2)\). In view of the considered symmetries in Fig. 1(c), the effective two-port system [44] is reciprocal. In other words, \( T \) should satisfy

\[ T_{11} = T_{22}, \]
\[ T_{11}T_{22} - T_{12}T_{21} = 1, \]
\[ v_0 = (A - B)/Z_0, \]
\[ v_d = (D - C)/Z_0. \]

(4)

(5)

(6)

(7)

(8)

(9)

Where \( p_0 \) and \( p_d \) are the pressures at locations \( x_0 \) and \( x_d \) in Fig. 1(a), respectively; \( v_0 \) and \( v_d \) express the particle velocities at locations \( x_0 \) and \( x_d \) in Fig. 1(a), respectively; \( Z_0 \) is the impedance of the background medium. Then the transfer matrix of the effective two-port network can be written as

\[ T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} p_{0\text{v}4} + p_{0\text{v}0} & p_{0\text{v}4} - p_{0\text{v}0} \\ p_{0\text{d}4} + p_{0\text{d}0} & p_{0\text{d}4} - p_{0\text{d}0} \end{bmatrix}. \]

(10)

Based on the transformation relation between the scattering and transfer matrices [44], the scattering matrix can be obtained as

\[ S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} T_{11} + T_{12}/Z_0 - T_{21}Z_0 - T_{22} & 2(-T_{21}T_{12} + T_{11}T_{22}) \\ T_{11} + T_{12}/Z_0 + T_{21}Z_0 + T_{22} & -T_{11} + T_{12}/Z_0 - T_{21}Z_0 + T_{22} \end{bmatrix}. \]

(11)

where \( S_{11} \) and \( S_{21} \) are the reflection \( R \) and transmission coefficient \( T \) respectively. If the size of the microstructure is much smaller than the operating wavelength of the background medium (i.e., \( \lambda_0 \geq 5a \)), the composite microstructure can be regarded as the homogeneous medium [45, 46], thus the effective impedance \( Z_{\text{eff}} \) and effective refractive index \( n_{\text{eff}} \) can be retrieved using the inverse technique [43] as

\[ Z_{\text{eff}} = \frac{\eta}{1 - 2R + R^2 - T^2}, \]
\[ n_{\text{eff}} = \frac{-i \ln \xi + 2\pi m}{k_0 a}. \]

(12)

(13)

where \( m \) represents the branch number of function \( \cos^{-1}[(1 - R^2 + T^2)/2T] \); \( \eta \) and \( \xi \) are defined by

\[ \eta = \pm \sqrt{(R^2 - T^2 - 1)^2 - 4T^2}, \]

(14)
\[ \zeta = \frac{1 - R^2 + T^2 + \eta}{2T}. \]

For passive metamaterials, the physically meaningful natural feature is that the sign of \( \eta \) should be chosen such that Re(\( Z_{\text{eff}} \)) is positive. The calculation of \( n_{\text{eff}} \) shown in Eq. (13) is highly dependent on the value of \( m \). For thick metamaterials, \( m \) should be carefully selected which usually takes a finite value as integer. For the sake of simplicity, the metamaterial can be constructed with a minimal thickness whose size is much smaller than the wavelength so that \( m = 0 \) can be guaranteed.

After \( Z_{\text{eff}} \) and \( n_{\text{eff}} \) have been determined, the effective mass density \( \rho_{\text{eff}} \) and bulk modulus \( K_{\text{eff}} \) are computed by

\[
\rho_{\text{eff}} = \rho_0 Z_{\text{eff}} n_{\text{eff}},
\]

\[
K_{\text{eff}} = \frac{\rho_0 c_0^2 Z_{\text{eff}}}{n_{\text{eff}}},
\]

where \( \rho_0 \) and \( c_0 \) are the mass density and acoustic velocity of the background medium, respectively. Meanwhile, the effective phase change \( \Delta \phi_{\text{eff}} \) across the metamaterial layer can be obtained by \( \Delta \phi_{\text{eff}} = \omega \rho_0 \rho_{\text{eff}} Z_{\text{eff}} \).

2.2. Design problem formulation

To realize a broadband double-negative AMM without membrane units, we need to construct a microstructure for resolving two emblematic challenges: (1) different resonance symmetries, including monopole, dipole and even quadrupole, have to be exploited through one microstructure; and (2) \( \rho_{\text{eff}} \) and \( K_{\text{eff}} \) should have the same dispersion property as frequency increases. Fortunately, as a systematic mathematical method, topology optimization involves the optimization of material layout in a huge design space \( \mathbb{R}^{N \times N} \), providing limitless possibilities for the occurrence of multiple resonances based on a brand-new topology. In general, the microstructure only holds single negativity \([47,48]\) or very narrow-band double negativity \([7,29]\) if the variation tendencies of \( \rho_{\text{eff}} \) and \( K_{\text{eff}} \) are inconsistent. Consequently, it is essential to control their holistic properties and then guarantee the coincident performance.

From the previous studies on AMMs \([7,47]\), we find that the metamaterial usually possesses a relatively large \( n_{\text{eff}} \) before generating the negative properties, no matter whether the metamaterial has single negativity \([47]\) or narrow-band double negativity \([7]\). More specially, the labyrinth microstructure has been demonstrated to be capable of achieving a long path length which is equivalent to a large \( n_{\text{eff}} \)[7]. As a result, the metamaterial can realize the band folding in the low-frequency range, thus creating the double negativity. For other types of AMMs \([47,48]\) with single negativity, the negative \( \rho_{\text{eff}} \) or \( K_{\text{eff}} \) requires a resonance to generate an infinite effective value. That is, \( \rho_{\text{eff}} \) will increase from a positive value to the negative infinity and then derive the negative values. However, \( K_{\text{eff}} \) will decrease from a positive value to the infinity. Hence, in pursuing double negativity, \( \rho_{\text{eff}} \) and \( K_{\text{eff}} \) separately can have the noticeable increasing and decreasing trends, respectively. In such a situation, the relatively large \( \rho_{\text{eff}} \) and relative small \( K_{\text{eff}} \) will allow the microstructure to possess a large \( n_{\text{eff}} \) over the entire spectrum of interest. Although \( n_{\text{eff}} \) will show the obvious dispersion property, its value can keep the quasi-static feature in the ultra-low frequency range. Therefore, regardless of the resonance mechanism and structural topology, the common prerequisite condition for double negativity is getting the relatively large \( n_{\text{eff}} \) in the ultra-low frequency range. When large \( n_{\text{eff}} \) induces the suitable resonances, it is essential to avoid the single negative \( \rho_{\text{eff}} \) or \( K_{\text{eff}} \) by adjusting and controlling the dispersion extent of \( \rho_{\text{eff}} \) and \( K_{\text{eff}} \). In other words, the optimization model must properly punish the degree of variation for \( \rho_{\text{eff}} \) and \( K_{\text{eff}} \) over the whole concerned spectrum while improving \( n_{\text{eff}} \) at the same time. Finally, to achieve broadband double negativity along the \( x \) direction within the target frequency ranges, a consolidated optimization formulation considering the prescribed physical mechanisms of effective constitutive parameters and special structural feature sizes is proposed as follows.

For: \( \omega \in [\omega_{\text{min}}, \omega_{\text{max}}] \)

Maximize: \( OF = N_0 + \frac{n_{\text{eff}}^{k+1} M - \alpha}{M} \times \max_{j = 1,2} \frac{\max(K_{\text{eff}}^{k+1})}{\min(K_{\text{eff}}^{k+1})} \)

Subject to: \( \rho_i = 0 \) or \( 1 \) (\( i = N \times N \)),

\( CD_{\text{air}} = 1 \).

\( \min(w_i) \geq w_s^* \)

\( \min(w_s) \geq w_s^* \)

\[ \beta = \begin{cases} 
\beta_1 = \min_{i = 1,2} \left[ \rho_{\text{eff}}^{k+2} - \rho_{\text{eff}}^{k+1}, \rho_{\text{eff}}^{k+1} - \rho_{\text{eff}}^{k-1} \right] > 0 & (\text{case 1}), \\
\beta_2 = \min_{i = 1,2} \frac{K_{\text{eff}}^{k+2} - K_{\text{eff}}^{k+1}}{K_{\text{eff}}^{k+1} - K_{\text{eff}}^{k-1}} > 0 & (\text{case 2}), \\
\beta_3 = \min_{i = 1,2} \frac{K_{\text{eff}}^{k+2} - K_{\text{eff}}^{k+1}}{K_{\text{eff}}^{k+1} - K_{\text{eff}}^{k-1}} \geq 0 & (\text{case 2}), \\
\beta_4 = \min_{i = 1,2} \frac{K_{\text{eff}}^{k+2} - K_{\text{eff}}^{k+1}}{K_{\text{eff}}^{k+1} - K_{\text{eff}}^{k-1}} \leq 0 & (\text{case 2}), \\
\end{cases} \]

where \( \omega_{\text{min}} \) and \( \omega_{\text{max}} \) are the upper and lower bounds of the target frequency range which is divided by \( M \) sampling frequency points; \( OF \) denotes the objective function; \( N_0 \) is the number of sampling frequency points where the double negativity is implemented; \( n_{\text{eff}}^s \), \( \rho_{\text{eff}}^s \) and \( K_{\text{eff}}^s \) are the arrays of positive \( n_{\text{eff}}^s \), positive \( \rho_{\text{eff}}^s \) and positive \( K_{\text{eff}}^s \), respectively; \( M' \) and \( N' \) are the numbers of the elements which belong to the arrays of \( \rho_{\text{eff}}^s \) and \( K_{\text{eff}}^s \), respectively; \( n_{\text{eff}}^{k+1} \) denotes the first positive effective refractive index of an array of \( n_{\text{eff}}^s \); \( \alpha \) is a prescribed parameter for regulating the whole dispersion extents of \( \rho_{\text{eff}}^s \) and \( K_{\text{eff}}^s \); \( \rho_i \) represents the material phase in optimization and declares the air (0) or solid (1) attribute of a pixel; \( CD_{\text{air}} \) denotes the number of the connected air domains in the microstructure; \( \Sigma \) stands for the design domain shown in Fig. 1(c). Here we employ the simple geometrical constraint in Eq. (21) based on the fact that multiple connected air domains usually reduce the wave transmission and will form several closed cavities, resulting in extremely narrow bandwidths of the resonances. Note that two special structural constraints in Eqs. (22) and (23) are introduced to ensure meaningful microstructures from physics and manufacturing perspective, respectively. Given that the AMMs with very narrow air channels usually incur the significant viscous losses which are induced by the near-wall viscosity effect \([47]\), it is necessary to restrict the dimensions of all air channels for circumventing this problem. More specifically, the minimum size of the array \( w_s \) composed by every air channel should be larger than a pre-set parameter \( w_s^* \). Furthermore, similar control over the solid
components is also needed for topology optimization, especially for satisfying the sufficient strength and fabrication requirement of the metamaterial samples [11,12,47]. Therefore, we set the constraint in Eq. (23) to cope with these two issues, i.e., the minimum size of an array \( w_0 \) including every solid must be larger than an empirical value \( w_\text{e}^* \).

Moreover, without taking particular control measures, inducing the overlapping resonances can easily bring about two different and inconsistent variations for \( r_\text{eff} \) and \( K_\text{eff} \), thus jeopardizing the formation of broadband double negativity. Therefore, we use the special physical constraint in Eq. (24) to precisely control the variation of \( r_\text{eff} \) and \( K_\text{eff} \) at the sampling frequency points. In terms of the discrete positive \( r_\text{eff} \) and \( K_\text{eff} \), their possible variation with the increase of frequency can be generalized into two major categories: one is the simultaneous increasing tendency (i.e., case 1); the other is the simultaneous non-increasing tendency (i.e., case 2). Throughout this paper, we will demonstrate the crucial role of these two mechanism constraints for inducing two novel double-negative microstructures.

Obviously, the optimization problem in Eqs. 18–24 involves different kinds of constraints, intensifying the difficulties of optimization search in a large design space. Many topology optimization methods can effectively solve the various structural optimization problems in different fields [49–52]. Here, owing to the strong versatility, the improved two-stage single-objective genetic algorithm (GA) [11,12,53,54] is utilized to solve the proposed optimization problem. GA treats the microstructure in \( N_1 \times N_1 \) pixels as a binary chromosome and mimics the evolutionary process by applying the natural selection principle to every generation towards the best design solution. First, an initial population of \( N_p \) individuals is randomly generated. To improve the effectiveness of any microstructures, a special “abrupt entropy filtering” [54] is applied for every microstructure to slightly fill up some isolated voids and remove some isolated elements. Secondly, every individual is evaluated for the fitness function and constrains. Then, GA uses the repetitive operators including the tournament selection, uniformed-matrix crossover and uniformed-matrix mutation to produce the offspring generation. Finally, the representative elitism strategy [53] is utilized to improve and accelerate the optimization. After the prescribed number of generations, GA produces the optimized individual at the first stage. Introducing the optimized individual as the “seed” structure with \( N_2 \times N_2 \) pixels, GA are performed through the corresponding genetic operators at the second stage. Repeat all procedures generation by generation, and then bring about the final optimized microstructure towards to optimization problem in Eqs. 18–24.

3. Results and discussions

In this section, the proposed topology optimization formulation in Eqs. 18–24 is applied to design the square-latticed subwavelength metamaterial to obtain the broadband double negativity within a target frequency range \([\Omega_{\text{min}}, \Omega_{\text{max}}]\). Three representative symmetries including the square, chiral and orthogonal cases are investigated to give the synthetical and thorough insight into the beneficial topological feature of microstructures. All optimizations reported in this paper start from a random initial population. We adopt the following mass densities and speeds of sound for the air and solid materials: \( \rho_{\text{air}} = 1.204 \text{ kg/m}^3 \), \( c_{\text{air}} = 343 \text{ m/s} \), \( \rho_{\text{solid}} = 1230 \text{ kg/m}^3 \) and \( c_{\text{solid}} = 2230 \text{ m/s} \) [55]. In fact, our numerical tests indicate that the acoustic-structural interaction has only a negligible effect on the performances of the effective parameters in the subwavelength range for the cases investigated in this paper. In general, the effects of the acoustic-structural interaction may have a visible difference between the “harder” and “softer” solids. For the analyses of the optimization performance, unless otherwise stated, the target frequency range is defined as \([100 \text{ Hz}, 8000 \text{ Hz}]\). The normalized frequency \( \Omega = \omega a / 2 \pi c \) is introduced for convenience, where \( a \) denotes the lattice constant, and \( c \) is the acoustic velocity of the air. The normalized target range is \([\Omega_{\text{min}}, \Omega_{\text{max}}]\) = [0.002476, 0.198061]. The number of sampling frequency points \( M \) is set as 11, which is suggested by the numerical tests considering the computing cost and effectiveness of the discrete description. The solid constrained parameter is selected as \( w_\text{e}^* = a / 30 \) in all optimizations. The parameters of the GA are the population size \( N_p = 30 \), the crossover probability \( P_c = 0.9 \), the mutation probability \( P_m = 0.02 \), and the championship selection size \( N_c = 18 \). At the first stage, the optimization is performed by 2500 generations in \( 30 \times 30 \) pixels. For the fine description of topologies, the optimization with other 2500 generations is executed in \( 60 \times 60 \) pixels. Meanwhile, the optimized microstructure in the first stage is introduced as an initial configuration in the second stage. All two-stage optimization processes are implemented within \( 34.5 \text{ h} \) on a Linux cluster with Intel Xeon X5650 Core @ 2.66 GHz. The numerical simulations of dispersion relations, effective parameters, eigenstates, transmission spectra and evanescent wave transmission are carried out by ABAQUS 6.14–1. Simulations of negative refraction and acoustic subwavelength imaging are accomplished by COMSOL Multiphysics 4.4. An acoustic experiment is conducted to demonstrate the subwavelength imaging of topology-optimized space-coiling AMM to exhibit the correctness of explored double-negative mechanisms and then to show the potential of our topology optimization framework.

3.1. Optimized double-negative AMMs under simultaneous increasing tendencies of the effective parameters

This subsection presents the optimization results with the prescribed simultaneous increasing tendencies \( \beta > 0 \) of the effective parameters, \( \rho_\text{eff} \) and \( K_\text{eff} \) (i.e., case 1 in Eq. (24)) for the normalized subwavelength target range \([0.002476, 0.198061]\). Some representative topological features, evolution history, various physical characterizations, negative properties of optimized metamaterials are analyzed and discussed in details. The typical and novel LC resonances contributing to the acoustic broadband double negativity are revealed through the resonance-cavity-based AMMs for the first time. All mentioned frequencies in the following contents refer to the normalized ones.

3.1.1. Topology-optimized resonance-cavity-based AMMs

3.1.1.1. Square, chiral and orthogonal symmetries. We firstly design the microstructure with square symmetry and explore the effects of \( a \) and air channel width on optimized topologies and double negativity, as shown in Fig. 2(a). Extracting their macroscopic geometry features, it is interesting to observe some common characteristics: (1) big air cavities connected through narrow air channels, and (2) big solid regions separated by the air domains. Like the Helmholtz resonator [2], multiple cavities in Fig. 2(a) can induce the negative effective bulk modulus. The distributions of hard solids can cause the large reflection with the limited space, which is beneficial for the occurrence of large refractive index. The above characteristics of resonance and large refractive index are dovetailed with the settings of the objective function in Eq. (19). With a larger \( a \), the metamaterial S2 has more cavities than S1. For the air-solid metamaterials with the viscous losses [47], the widths of air channels have appreciable impact on the efficacy of metamaterials. Fortunately, increasing this feature size can admittedly reduce the viscosity factor of metamaterials. To show the effect of air channels, we illustrate the optimized metamaterial S3 for the
typical feature sizes of $w_a = a/15$ in Fig. 2(a). Compared with S1, S3 has the similar topological features except four additional slender hard solid plates. For clearly showing the desired negative properties, we present in Fig. 2(d) the double-negative ranges and quasi-static refractive index and impedance of the AMMs in Fig. 2(a). Since the effect of $a$ is non-monotonic, we can only suggest its suitable range of [1.0, 1.5] in which double negativity can be effectively realized. The relatively larger refractive index usually enables a relatively wider double negativity. Combining the microstructure topologies, the variation of impedance shows that a reduction of air cavity domains can cause an increase of the impedance. Hence S1 can keep the relatively large refractive index while maintaining the reasonable wave transmission. The difference between S1 and S2 shows that the superabundant cavities may result in the narrow-band double negativity. The difference between S1 and S3 suggests that wider air channels will evoke smaller regions of air cavities, ultimately leading to a narrow-band double negativity.

To check the effectiveness of the proposed optimization formulation, we further investigate the topology optimization with chiral and orthotropic symmetries, see Fig. 2(b) and (c). Fig. 2(d) displays their double-negative ranges and quasi-static refractive indices and impedances. All chiral metamaterials in Fig. 2(b) contain a large air cavity in the center and four rotationally distributive solid blocks. For the unit-cell domains marked by dashed lines, four corner regions of the metamaterial can be regarded as four small air cavities. With the same $w_a$, S5 has double negativity property than S4 and S6. So $a = 1.0$ is effective to balance the large refractive index and appropriate dispersion extent. Unlike the square-symmetry case in Fig. 2(a), the topology of S7 demonstrates that large $w_a$ prescribed in optimization can naturally make the solid components thinner. Moreover, the chiral symmetry is superior to the square symmetry if the complexity of structure is ignored. The double-negative ranges of optimized chiral metamaterials are apparently larger than those of the square-symmetry ones in Fig. 2(a). Therefore, multiple air cavities combined with the zigzag air channels and solid parts provide the ideal geometrical platform for the broadband double negativity.

Fig. 2(c) presents the optimized orthogonal-symmetry metamaterials. For the subwavelength imaging, it is essential to make sure that the effective bulk modulus is isotropic during optimization for all potential designs. Although the effective bulk modulus is normally isotropic if the operating wavelength is larger than 5$\lambda$ (recall that $\lambda$ is the lattice constant) [45,46], many highly complex orthogonal-symmetry metamaterials in optimization may have distinct behaviors along two principle directions, resulting in the high anisotropy and even possible coupling of effective bulk modulus using the present simulation model in Fig. 1(a). As a remedy approach, we force the relative difference between $K_{\text{eff}}$ retrieved from the $x$ and $y$ direction wave simulations to be smaller than 5%. More performances are summarized in Appendix A.
Through comparing the results in Fig. 2(d) for the three cases, we can make the following observations. For the low-frequency property, the optimized AMMs show good double negativity, with chiral symmetry being the best followed by orthogonal symmetry. The behaviors of refractive index and impedance are positively correlated. The orthogonal-symmetry AMMs can realize the similar refractive index with the chiral-symmetry ones. But the chiral-symmetry AMMs have to face the relatively large impedance. Comparing the double-negative ranges of S4–S7, we find that the effect of $\alpha$ is smaller than that of the minimal air channel width. Similar feature can be observed from the orthogonal-symmetry case. Therefore, the control over the feature size of the air channels should be a pivotal factor in designing the double-negative AMMs for practical applications. Given the same topological features, increasing the air channel widths will result in a decrease of the refractive index and impedance for the optimization under simultaneous increasing tendencies. For the chiral symmetry, the relatively large (1.5) or small (0.5) $\alpha$ will result in a smaller refractive index. However, for the orthogonal symmetry, larger $\alpha$ can lead to a larger refractive index for double negativity.

Based on the solid-air system, the present topology optimization can effectively realize the novel multi-cavities microstructures having ideal double negativity, and overcome the limitations of single negativity of the Helmholtz metamaterials [2,25]. From the prospective of double negativity, introducing the chirality is the best design approach; followed by the orthogonal symmetry and the square symmetry. Similarly, the chiral symmetry can induce the largest refractive index. From the prospective of topological features, three symmetric AMMs share the common ground for double negativity: multiple air cavities, solid blocks and relatively narrow air channels.

3.1.1.2. Analysis of representative AMM S1. In view of the most concise topological features and satisfactory double negativity, the AMM S1 is suitable to be systematically analyzed as the representative metamaterial. To clarify the evolution for the optimized topology of S1 in Figs. 2 and 3 shows the evolutionary history of the maximal fitness with the generation number during the “coarse to fine” optimization process. Topology optimization starts from a randomly generated microstructure ($G = 0$), which cannot satisfy the particular constraints of Eqs. 21–24. From the generation $G = 20$ ($F = 0.1066$) to $G = 215$ ($F = 0.2563$), GA can quickly capture the beneficial topological feature during the early evolution stage, i.e., four air cavities and two solid blocks. The maximal fitness change between $G = 215$ ($F = 0.2563$) and $G = 320$ ($F = 2.6097$) implies that the small central solids can contribute to the formation of double negativity. From the generation $G = 320$ ($F = 2.6097$) to $G = 3435$ ($F = 3.7124$), the microstructure turns to possess the larger air channels at four corners between the air cavities. Furthermore, the microstructure obtains the clearer edge descriptions and more smooth geometrical layouts. From $G = 3435$ ($F = 3.7124$) to $G = 4276$ ($F = 3.7213$), the slightly increased fitness demonstrates that the larger cavities should be a better choice under the circumstance with unchanged air channels. Therefore, we can generalize the beneficial topological features for the square-symmetry AMMs: four large enough air cavities, two independent solid blocks, narrow enough air channels and several slender hard solid plates.

To systematically characterize S1, we show in Fig. 4 the corresponding dispersion relations, effective constitutive parameters, wave transmission property and pressure magnification. Band structure in Fig. 4(a) displays a single band of $0.156384, 0.226167$ with negative curvature. The near-linear trait of the first band indicates the homogenous wave behaviors in the deep-subwavelength scale. The effective parameters in Fig. 4(b) show the simultaneous negative properties for $n_{\text{eff}}^s$ and $K_{\text{eff}}$ within the negative band range illustrated in Fig. 4(a). The positive values of $\rho_{\text{eff}}$ and $K_{\text{eff}}$ increase simultaneously at the established sampling frequency points (i.e., hollow circles and triangles). Indeed, their variations are consistent with the constraints imposed by Eq. (24). More importantly, $n_{\text{eff}}^s$ and $Z_{\text{eff}}^s$ have relatively large values and keep the simultaneous increasing variation as well. Negative $n_{\text{eff}}^i$ is generated within the same range as double-negative band in Fig. 4(a). There is a complete bandgap above the negative band, confirming the zero values of Re ($n_{\text{eff}}^i$). Using the effective parameters, we can obtain in Fig. 4(a) the retrieved dispersion relations which perfectly match the band structures. The transmission spectrum in Fig. 4(d) shows that regardless of the thickness of the AMM, the total transmission always appears near the lower edge of the double-negative range. In view of the dramatic change in Re ($Z_{\text{eff}}^s$) near the lower-edge frequency, the perfectly-matched effective impedance Re ($Z_{\text{eff}}^s = 1$) should be the physical origin of the total transmission. When the number of unit-cell increases ($N = 1, 2, 10$), the Fabry-Perot resonance conditions can be satisfied at more frequencies, consequently causing more standing waves with high transmission compressed within the AMM.

Subsequently, wave transmission based on a microstructure S1 is calculated to demonstrate the essential resonances, see Fig. 4(e). Clearly, the localized pressure generates two peaks with the increase of frequency, confirming the pressure magnification in the region of the upper cavity. To include the influence of the viscous-thermal losses on the material performance, we adopt the simplified equivalent model in Ref. [47] for simplicity. More specifically, we add the loss explicitly into the wavenumber of air as $k_0 = \omega/\gamma_{\text{air}} - \gamma_{\text{loss}} \times \omega^2/c_{\text{air}}$, where $\gamma_{\text{loss}}$ is the loss factor. The simulations with the loss factor of 0.004, 0.0093 and 0.022 are performed and depicted in Fig. 4(e). It is obvious that S1 can strike a good balance between the resonance transmission and immunity to dissipation losses.
It is noticed here that we did not adopt the sophisticated thermal-acoustic model considering the viscous-thermal losses for the metamaterials with the Fabry-Perot resonances in Refs. [56–58]. This is based on the fact that the double negativities of the present metamaterials are mainly induced by the overlapping local resonances other than the Fabry-Perot resonances. Note that the specific resonance mechanisms will be analyzed and discussed in the following section.

3.1.2. Mechanisms of the optimized double negativity

To understand the physics of the double negativity in the resonance-cavity AMMs, we systematically study the eigenstates in the bands of $S_1$ to $S_6$, $S_9$, and $S_{13}$. Fig. 5 shows that optimized AMMs support typical LC resonances to guarantee the overlapping of different multipolar (monopolar, dipolar and quadrupolar, etc.) resonances. The eigenstates $MS_1$ and $KS_1$ give the understanding about the physical origin of negative band shown in Fig. 4(a). To be specific, $MS_2$ shows the clear dipolar resonance caused by the highly localized energy in the left and right cavities. The infinite value of the effective mass density in Fig. 4(b) indicates that $MS_2$ is responsible for the negative effective mass density. As for $KS_2$, most energy is localized in four cavities, forming the quadrupolar resonance and causing the infinite effective bulk modulus. Meanwhile, the negative value of $K_{eff}$ in Fig. 4(b) confirms the negative bulk modulus produced by $KS_2$. In this case, the combination of the dipolar and quadrupolar resonances can generate the double negativity. Since the range of negative effective bulk modulus is smaller than that of the negative effective mass density, the range of the negative effective bulk modulus dominates the bandwidth of double negativity. Similarly, eigenstates $MS_2$ and $KS_2$ also clearly exhibit dipolar and quadrupolar resonances with different locations and occupied spaces of localized energy, respectively. Because the double negativity originates from the overlapping resonances, the double-negative range is determined by the common spaces which can support two kinds of resonances. Consequently, $S_2$ has a smaller double-negative range than $S_1$. Based on the same principle, the resonance space of eigenstate $KS_3$ implies that $S_3$ should have the smallest double-negative range for the square-symmetry case. Unlike the square-symmetry cases, however, $MS_5$ and $KS_5$ show the quadrupolar and hybridization of quadrupolar and monopolar resonances for the negative effective mass density and bulk modulus, respectively. Apparently, their overlapping enlarges the double-negative range. Eigenstate $KS_6$ also shows similar hybridization effect only with different chiral resonance spaces.
Nevertheless, MS9 and KS9 indicate that the orthotropic symmetry mainly alters the locations and topologies of the four cavities in the topology optimization, rather than the forms of resonances for double negativity.

To reveal the above LC-resonance mechanisms, we take S1 and S5 in Fig. 2 as examples to illustrate their equivalent physical models ES1 and ES5 in Fig. 5, respectively. In fact, an acoustic resonator is analogous to an inductor-capacitor circuit in terms of resonance properties [2], whose enclosed cavity acts as a capacitor; and relatively narrow air channels as inductor. Obviously, the optimized resonance-cavity-based AMMs can be regarded as a combined system comprising several inductor-capacitor circuits. When pressure variation occurs in the channels, the distributions of the inductors and capacitors determine the excited forms of LC resonances. In other words, the model ES1 can support the eigenstates MS1 and KS1 under two kinds of excitations, yielding the infinite effective mass density. However, the model ES5 has different topological feature, i.e., four capacitors locate in the corners and one capacitor in the center. On one hand, this distribution can induce the quadrupolar resonance for negative effective mass density. On the other hand, if five capacitors are excited simultaneously, the hybridization of quadrupolar and monopolar resonance can arise from model ES5 as well.

3.2. Optimized double-negative AMMs under simultaneous non-increasing tendencies of the effective parameters

This subsection presents the optimization results with the prescribed simultaneous non-increasing tendencies \( \beta \leq 0 \) for both \( \rho_{\text{eff}} \) and \( K_{\text{eff}} \) (i.e., case 2 in Eq. (24)). To show the great potential of “non-increasing” mechanism, the optimization uses the more strict geometrical constraint, i.e., \( w^*_a \) is set as \( a/15 \). Similarly, some representative topological features, evolution history, various physical characterizations, negative properties of the optimized metamaterials are analyzed and discussed in details. The emblematic and novel Mie resonances causing the acoustic...
broadband double negativity are perfectly revealed through the space-coiling AMMs.

3.2.1. Topology-optimized space-coiling AMMs

3.2.1.1. Square, chiral and orthogonal symmetries. Owing to the general feature of the proposed optimization framework in Eqs. 18–24, we apply it to design the broadband double-negative AMMs under the simultaneous non-increasing tendencies ($\beta \leq 0$) considering square, chiral and orthotropic symmetries, see Fig. 6. Interestingly, these AMMs exhibit the common topological features essentially different from those shown in Fig. 2: (1) several separated solid blocks in the coiling up space, (2) zigzag air channels forming the labyrinth layouts, and (3) a number of local air regions. Intuitively, like the extreme metamaterials reported by Liang et al. [7], waves can freely propagate inside the curled space with materials of negligible loss, arousing the large phase delays within a small space and then realizing the large refractive index. Consequently, the band folding [7] supporting the double negativity should emerge at the subwavelength scale. Compared with the optimized space-coiling topology for generating double negativity. The comparison of S12 and S13 shows that increasing $\alpha$ can reduce the air paths and induce the thinner hard solid plates. Since S14 is generated with the different target spectra, similar topology and double negativity of S14 demonstrate that the proposed optimization strategy is robust for specific frequencies. Results in Fig. 6(b) also show that wide air regions, except the interconnection core regions, and relatively thick hard solid plates can result in the degenerations of double negativity. Therefore, the most beneficial space-coiling topology should include suitable zigzag channels, thin curved hard solid plates and interconnection core regions in the center.

To further check the negative properties of the space-coiling AMMs in Fig. 6(a) and (b) presents their double-negative ranges and quasi-static refractive index and impedance. For the low-frequency performance, the optimized AMMs, S12, S14, S15 and S16, have the conspicuous broadband double negativity transcending the previous extreme metamaterials [7]. Moreover, all optimized space-coiling AMMs in Fig. 6 possess larger air channels than those of the previous extreme metamaterials [7,46], thus obviously reducing the viscous loss. In spite of the more strict constraints on air channels than those in Fig. 2, the optimized AMMs in Fig. 6 can also provide the desired negative bands. With the same symmetry, the variations of the impedance are positively related to refractive index. Differences between S12, S14, S15 and S16 show that the orthotropic-symmetry AMMs can generate similar refractive index with the chiral-symmetry ones. However, the corresponding impedance will be obviously increased. This further illustrates that the chiral symmetry can result in ideal double negativity and wave transmission simultaneously. For the chiral symmetry, the difference between the performance of S12 and S13 shows that a relatively large $\alpha$ can induce a smaller refractive index and impedance, causing a relatively small double-negative range. In the other words, the variation of the refractive

![Fig. 6. Topology-optimized space-coiling AMMs with three representative symmetries. (a) Optimized microstructures. All topology-optimized metamaterials are under the constraint of $\beta \leq 0$. The target spectrum of S14 is selected as [0.09903, 0.198061] (i.e., $\omega' = 0.09903$. The ranges of the other structures are set as [0.002476, 0.198061]. For the microstructures S15 and S16, to guarantee the isotropy of effective bulk modulus, the relative difference between $K_{\text{eff}}$ retrieved from the x direction and y direction wave simulations are forced to be smaller than 5%. (b) Comparisons of double negativities, the quasi-static effective refractive index $n_{\text{eff}}^{-1}$ and impedance $Z_{\text{eff}}^{-1}$ at $\omega_{\text{min}}$ for S11–S16. More performances are summarized in Appendix A.](image-url)
reported by Liang et al. [7], the AMM S12 shows the emblematic core region in the center maintains a relatively large size. In spite of curved portion of the zigzag paths increases, thus producing a (for getting double negativity. From G making channels and solids parts more curved is an effective way of double negativity at generation G solid plates and interconnection core regions. The creation of the search ability of the GA in determining the separated thick hard number for the AMM S12 in Fig. 7. The noteworthy change from the evolutionary history of the maximal fitness with the generation coarse to fine (G = 1.5521) to G = 10 (F = 0.2859) illustrates the strong searching ability of the GA in determining the separated thick hard solid plates and interconnection core regions. The creation of the double negativity at generation G = 117 (F = 1.5521) suggests that making channels and solids parts more curved is an effective way for getting double negativity. From G = 117 (F = 1.5521) to G = 4680 (F = 4.0818), the solid components become thinner while the curved portion of the zigzag paths increases, thus producing a bigger double negativity range. Meanwhile, the interconnection core region in the center maintains a relatively large size. In spite of the similar space-coiling topology with the extreme metamaterials reported by Liang et al. [7], the AMM S12 shows the emblematic features of larger interconnection regions and more curved degree of the hard solid plates, which gives rise to better double-negative property.

To characterize the double negativity of S12, we systematically study the dispersion relations, effective parameters and transmission spectrum, see Fig. 8. For the second band range displayed in Fig. 8(a), the different physical quantities coincide mutually. Interestingly, owing to the band folding, the slopes around the Γ point in both the ΓX and ΓM directions are almost the same in the first, second, forth, sixth, eighth and tenth bands. In fact, this band folding not only indicates the isotropic indices, but also gives rise to the ideal negative properties at both subwavelength and long-wavelength regimes. Just as the constraints of simultaneous non-increasing tendencies in Eq. (24), the positive ρ_eff and K_eff decrease simultaneously below the negative range. The simultaneous negative ρ_eff and K_eff in Fig. 8(b) and negative ρ_eff clearly demonstrate the double-negative property of the negative band depicted in Fig. 8(a). Unlike the results in Fig. 8(b), the positive n_eff and Z_eff have the opposite variation patterns. Using the effective parameters in Fig. 8(b) and (c), the retrieved dispersion relations based on EMT can perfectly match the band structures. Benefitting from the perfectly-matched effective impedance Re (Z_eff) = 1, total transmission appears near the lower edge of the double-negative range no matter whether the metamaterial layer is thin (N = 1-2) or thick (N = 10), see Fig. 8(d). Besides, the high transmission at other frequencies can also be obtained because of the satisfied Fabry-Perot resonance conditions.

The wave transmission based on a microstructure of S12 is further calculated to demonstrate the essential resonances, as shown in Fig. 8(e). Clearly, large magnitude pressure is mainly localized in the central interconnection region, showing a pressure magnification in the region of the upper cavity. Obviously, S12 allows striking a good balance between the resonance transmission and immunity to dissipation losses. Note that the specific

3.2.1.2. Analysis of representative AMM S12. To understand the origin of the space-coiling topology shown in Fig. 6, we show the evolutionary history of the maximal fitness with the generation number for the AMM S12 in Fig. 7. The noteworthy change from the generation G = 0 (F = 0) to G = 10 (F = 0.2859) illustrates the strong searching ability of the GA in determining the separated thick hard solid plates and interconnection core regions. The creation of the double negativity at generation G = 117 (F = 1.5521) suggests that making channels and solids parts more curved is an effective way for getting double negativity. From G = 117 (F = 1.5521) to G = 4680 (F = 4.0818), the solid components become thinner while the curved portion of the zigzag paths increases, thus producing a bigger double negativity range. Meanwhile, the interconnection core region in the center maintains a relatively large size. In spite of the similar space-coiling topology with the extreme metamaterials reported by Liang et al. [7], the AMM S12 shows the emblematic features of larger interconnection regions and more curved degree of the hard solid plates, which gives rise to better double-negative property.

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resonance mechanisms will be analyzed and discussed in the following section.

3.2.2. Mechanisms of optimized double negativity

To reveal the origin of the double negativity in the space-coiling AMMs, we take four metamaterials S12–S15 as examples and scrutinize their specific eigenstates in Fig. 9. Obviously, eigenstates MS12 and KS12 portray the quadrupolar and hybridization of quadrupolar and monopolar Mie resonances which can essentially induce the negative effective mass density and bulk modulus, respectively. It is relevant to note that these artificial Mie resonances distinctly differ from the LC resonances shown in Fig. 5. In principle, the Mie resonance usually appears in the structure having a high refractive index relative to the background medium [27,47]. Instead of highly localizing all energy within several cavities, the Mie resonances enable energy concentration in the air regions while producing apparent radiation, showing the feature of resonance scattering. Eigenstates MMS12 and KKS12 behave like a second-order quadrupolar and hybridization of the quadrupolar and monopolar Mie resonances, and characterize the double-negative essence of the sixth band in Fig. 8(a). Similarly, the high-frequency negative band (tenth band) in Fig. 8(a) also has the double negativity resulted from the higher-order Mie resonances. Eigenstates MS13 and KS13 also show that the double negativity of S13 is generated by the quadrupolar and hybridization of the quadrupolar and monopolar Mie resonances. Comparing the field distributions of MS12, KS12, MS13 and KS13, we can infer that the relatively large interconnection core region will cause the double negativity in the lower frequency range, see the negative ranges of S12 and S13 shown in Fig. 6(b). In addition, eigenstates MS14 and KS14 show that the Mie resonances can also be induced by more complex labyrinth structures. This means that the space-coiling topology is very robust for generating the multipolar Mie resonances. In addition, eigenstates MS15 and KS15 show that the ortho-symmetric labyrinth topology is conducive to similar Mie resonances as well. Hence the AMMs presented in Fig. 6 provide unanticipated topological features for both Mie resonances and double negativity. Note that the bandwidth of the double negativity is determined by the size of the overlapping regions for two resonances. However, in case of the quadrupolar and hybridization of the quadrupolar and monopolar Mie resonances, the largest overlapping regions should be the four corner areas, which indicates the corresponding limited bandwidth. In particular, it is the combination of multipolar resonances that provide the broadband double negativity over the previous studies on space-coiling AMMs [7]. Overall, eigenstates in Fig. 9 disclose that the optimal mechanism
with space-coiling topology for broadband low-frequency double negativity should be the combination of quadrupolar and hybridization of quadrupolar and monopolar Mie resonances.

To reveal the above Mie-resonance mechanisms, we also present in Fig. 9 the equivalent physical models ES12 and ES13 of the AMMs S12 and S13, respectively. Because S12 has the high effective refractive index and an air cavity in the center, the whole microstructure can be equivalent to four channels composed of the ultraslow medium connected with an air interconnection core. And they are separated by the solid frame materials. When waves propagate in the four channels with different phases, the model ES12 can produce the quadrupolar Mie resonance MS12 or the hybridization of the quadrupolar and monopolar Mie resonance KS12. In particular, the relatively large interconnection core further promotes this hybridization. Since the holistic effective refractive index of S13 is smaller than that of S12, the corresponding equivalent model ES13 has different straightened channels while ensuring similar geometrical feature. In addition, since S13 has smaller air cavities in the center than S12, ES13 should have smaller air interconnection core as well. As a result, the difference between ES12 and ES13 mainly affects the frequency range of double negativity.

3.3. Brief summary on two categories of AMMs

Due to the tremendous inverse-design ability, topology optimization has explored two categories of novel AMMs with broadband double negativity, namely the resonance-cavity-based and space-coiling metamaterials, respectively. For the desired negative properties, the space-coiling metamaterials can realize a wider double negativity within the lower frequency ranges, showing superiority over the resonance-cavity-based ones. In addition, the double negativity of the space-coiling metamaterials is less affected by the width of the air channels. And space-coiling metamaterials can produce the double negativity by introducing larger air channels. A good resonance-cavity-based metamaterial should possess the multiple air resonance cavities, several hard solid plates and air channels. For the space-coiling metamaterials, the beneficial
topologies should include the suitable zigzag channels, thin curved hard solid plates and interconnection core regions in the center. For the double-negative mechanisms, the resonance-cavity-based metamaterials benefit from the novel multipolar LC resonances. But the space-coiling metamaterials support the novel multipolar Mie resonances. In principle, both novel mechanisms overcome the limitations of the reported negative resonances \[ \{2,7,25,46,47\} \], representing the optimal physical essences for broadband double negativity so far. Because of the most straight hard solid plates, the space-coiling metamaterials are easier to be manufactured than the resonance-cavity-based ones. Anyway, the topology-optimized AMMs presented in this paper can achieve the double negativity in a brand-new structural style.

3.4. Potential applications using topology-optimized double-negative AMMs

This subsection presents the numerical results of broadband double negativity and the enhancement of evanescent wave transmission for LC-resonance and Mie-resonance optimized AMMs. Then we respectively show the negative refraction and acoustic subwavelength imaging with high transmission. Finally, the experimental demonstration of subwavelength imaging is successfully realized.

3.4.1. Numerical demonstrations of wave behaviors

To demonstrate the desired negative dispersions, we illustrate the equi-frequency surfaces (EFSs) of metamaterials S1 and S12 in Fig. 10(a) and (b), respectively. It is noted that two negative bands show the quite isotropic behavior within the whole range except the lower edge of the bands with the slight anisotropy. In addition, we can clearly observe the striking difference between the LC-resonance and Mie-resonance negative bands in Figs. 4(a) and 8(a), i.e., two bands are arc-shaped and nearly-straight, respectively. The variations of the EFSs with the increasing frequency in Fig. 10(a) and (b) suggest that the negative group velocities should occur along all directions. When waves are incident to the interface between the AMMs and background media, the refracted group velocity should be pointed to the direction of frequency increasing which is perpendicular to the contours, causing the expected negative direction. It is worth mentioning that the target frequency spectrum of \[ \{0.002476, 0.198061\} \] can guarantee the all-angel negative refraction for the whole target frequency spectrum. Then the subwavelength imaging can be realized within the whole target frequency spectrum as well.

To validate the negative refraction and subwavelength imaging, we display the corresponding simulation results for the metamaterials S1 and S12 in Fig. 11. As predicted in Fig. 11(a) and (c), when the Gaussian beam (45°) of an acoustic wave is incident from the left region, the desired negative refraction with high transmission can be clearly observed at \( \Omega = 0.173303 \) and \( \Omega = 0.160925 \), respectively. Since a recent study found that the acoustic-structural interaction may have a certain effect in the space-coiling AMMs \[58\], we display the numerical simulations with considering the acoustic-structural interaction in Fig. 11(b) and (d). The nearly identical beam patterns demonstrate that the acoustic-structural interaction has only a very slight effect on the wave propagation properties of the optimized AMMs in this study. Meanwhile, when a point source is excited in the left of the metamaterial slab, obvious imaging effect for S1 and S12 can be observed in the exiting surfaces of the slabs in Fig. 11(e) and (f). Their full widths at the half maximum (FWHM) of images are 0.44λ and 0.39λ, respectively, which are beyond the diffraction limit. Therefore, topology-optimized metamaterials S1 and S12 are demonstrated to have the ability of realizing the subwavelength imaging. In addition, we also present the simulations for S12 at the different frequencies in Fig. 11(f)-11(i). Comparisons between the results in Figs. 11(f)-11(i) show that the metamaterial lens can effectively generate the stable subwavelength imaging within the negative range. With the increase of frequency, the imaging resolution decreases and more energy is reflected by the lens due to the inevitable impedance mismatch.

To establish the subdiffraction-limit resolution in Fig. 11, we present in Fig. 12 the zero-order transmission coefficient \[ \{2,23,24\} \] of a plane wave for evaluating the transmission of both propagating and evanescent waves through an 8-layered metamaterials of S1 or S12 immersed in air. Note that a value of zero-order transmission larger than 1.0 implies the enhancement of the propagating or evanescent waves. The regions representing the propagating waves locate in the left of the skew lines in Fig. 12(a) and (b). And the regions located in the right of the skew lines describe the case of evanescent waves. For every frequency within the double-negative range, it is clear that the transmission coefficient can be larger than 1.0 for the wave vector \( k_y \) either near or far away from \( k_0 \). And the lower-frequency range has the enhancements in the wider range of \( k_y \) than the higher-frequency one. This emphasizes the importance

Fig. 10. Equi-frequency surfaces of topology-optimized AMMs. Surface plots of the first negative band (the second band) over the whole Brillouin zone for S1 (a) and S12 (b).
Fig. 11. Simulations of negative refraction and acoustic subwavelength imaging using topology-optimized AMMs. (a)–(d) Pressure fields under an incident Gaussian beam (45°) of acoustic waves for S1 without (a) or with (b) considering the acoustic-structural interaction at \( U = 0.173303 \) and S12 without (c) or with (d) considering the acoustic-structural interaction at \( U = 0.160925 \), respectively. (e) Imaging field pattern and intensity profiles (the source is located at the position \( a \) away from the left side of the 32 × 8 metamaterial slab) for S1 at \( U = 0.173303 \) (FWHM = 0.361). (f)–(i) Imaging field patterns and intensity profiles for S12 at \( U = 0.160925 \) (FWHM = 0.391) (f), 0.173303 (FWHM = 0.361) (g), 0.185682 (FWHM = 0.421) (h) and 0.198061 (FWHM = 0.441) (i), respectively.

Fig. 12. Enhanced transmission of the evanescent waves through topology-optimized AMMs. Frequency and wave-vector dependence of the zero-order transmission coefficient for both propagating and evanescent waves through a layer consisting of 8 metamaterials S1 (a) or S12 (b). Two skew lines in (a) and (b) represent the dispersion curves for air. If \( k_y \leq k_0 \) (where \( k_0 \) is the propagation constant of the fundamental waveguide mode), the transmission coefficient characterizes the transmission property for the propagating waves, while for \( k_y \leq k_0 \) the corresponding waves represent the evanescent waves.
of the frequency in enhancing evanescent waves for imaging. Consequently, benefiting from the enhancement of evanescent waves, the metamaterial lens can capture the subwavelength information of the object and then transfer the corresponding energy to the focal plane of the image.

3.4.2. Experimental verification of acoustic subwavelength imaging

Above results show that the space-coiling AMMs can realize the ideal broadband double negativity with relatively large air channels. In addition, they are mainly composed of straight hard solid plates, showing good workability. To show the convincing potential of the optimized AMMs, we worked with the space-coiling AMM S12 and experimentally demonstrated the broadband subwavelength imaging in Fig. 13. We adopted 3D-printing to fabricate an AMM sample made of polylactide acid (PLA) with a mass density of 1250 kg/m$^3$ and bulk modulus 3.5 $\times$ 10$^5$ Pa. The fabricated metamaterial slab in Fig. 13(a) and (c) consists of 20 $\times$ 5 microstructures depicted in Fig. 13(b). Experimental apparatus for the acoustic experiment inside a waveguide is illustrated in Fig. 13(a) where the slab sample was surrounded by the acoustic absorbing foams to avoid reflections. A loudspeaker, located 3 cm away from the input interface of the slab, was used as the point source of waves; while the mounted microphone measured the acoustic pressure by moving in the scanning area. Signals at each position were averaged over four measurements. Using the Fourier transform, the whole acoustic filled was obtained after the scanning measurement.

We first tested the subwavelength imaging of the metamaterial slab at 2200 Hz which is within the double-negative frequency range. The measured results in Fig. 13(e) agree well with the simulation results in Fig. 13(d) in terms of the acoustic magnitude for the dashed area. The measured results at 2350 Hz in Fig. 13(f) also show the desired imaging pattern. The measured imaging resolutions of Fig. 13(e) and (f) are 0.38$\lambda$ and 0.44$\lambda$, respectively, certifying the subwavelength property well. Then we study the performance of the subwavelength imaging within the range of [1700 Hz, 2500 Hz], as displayed in Fig. 13(g). One peak with high transmission can be clearly observed at all measured frequencies, demonstrating the broadband characteristic of the subwavelength imaging. Moreover, the non-monotonic curve in Fig. 13(h) exhibits all imaging resolutions less than or equal to the diffraction limit. Overall, the low operating frequency is beneficial to the high resolution. Clearly, the measured subwavelength imaging in Fig. 13 is attributed to the double negativity of the metamaterials.

4. Conclusions

In summary, for the first time, we construct a unified topology optimization framework for systematic designing the double negativity with any manual requirement including the expected microstructure symmetry, derivable double-negative mechanisms, necessary structural feature sizes and dispersion extent control of effective parameters. We design various novel microstructures with broadband double negativity and reveal the most beneficial topological features of resonance-cavity-based and space-coiling metamaterials. One feasible design principle is the suitable assembling of multiple air resonant cavities, solid blocks and air channels for resonance-cavity-based structures. Alternatively, one can realize suitable combinations of zigzag channels, thin curved hard solid plates and interconnection core regions in the center. Exhaustive characterizations of the metamaterials indicate that the double negativity, originating from the novel multipolar LC or Mie resonances, can be induced by the simultaneous increasing or non-increasing mechanisms in optimization. Desired acoustic negative refraction and subwavelength imaging of the optimized AMMs are numerically demonstrated in details for two representative AMMs. The enhancements of evanescent waves propagating through the metamaterials are found to be responsible for the subdiffraction-limit imaging resolution. In addition, we also experimentally validate the broadband subwavelength imaging of the space-coiling AMMs.

More importantly, the proposed topology optimization framework, involving the derivable LC and Mie resonances, is not restricted to the double negative metamaterials presented here. In principle, the design strategy proposed here can be universal and applicable to other types of AMMs demanding negative constitutive parameters, no matter whether they are double-negative [26–30], single-negative [23,24,40] or even hyperbolic [23,24,35]. The present optimized AMMs and superlens provide the subwavelength imaging with powerful and heuristic components, pushing the conceptual design to the specific practical applications. Our future work will focus on the in-depth design and realization of three-dimensional double-negative AMMs by topology optimization.

Finally, we would like to mention that the present optimization excludes the effect of the viscous-thermal losses. Although our experimental results show that the double negativities are not visibly affected by the viscous-thermal losses, at least for the samples used in our experiments, the effect of the viscous-thermal losses on topology optimization is an interesting topic for the future work.

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Appendix A. Supplementary data

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References
