Characterization of Nonplanar Second Harmonic Lamb Waves With a Refined Nonlinear Parameter

Structural health monitoring (SHM) methods based on the cumulative second harmonic Lamb waves show attractive advantages. An ideal nonlinear parameter should allow precise characterization of the cumulative effects of the distributed nonlinear sources such as the material nonlinearity of a plate (MNP), in the presence of other unavoidable localized nonlinear components. While highlighting the deficiencies of the traditional nonlinear parameter (TNP) in the nonplanar cases, a refined nonlinear parameter (RNP) is proposed. Through compensations for the wave attenuation associated with the wave divergence, the new parameter entails a better characterization and differentiation of the cumulative MNP and other noncumulative localized nonlinear sources. Theoretical findings are ascertained by both finite element (FE) simulations and experiments, through tactically adjusting the dominance level of different nonlinear sources in the system. Results confirm the appealing features of the proposed RNP for SHM applications.

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where $A_{NL}(2\omega)$ and $A_{\alpha}(\omega)$ are the amplitudes of the second harmonic and fundamental Lamb waves, respectively. The amplitude of the wave components can be obtained from either displacement or any other sensor output [25]. Whatever is used, the relative nonlinear parameter allows tracking the changes of material status, in compliance with the SHM philosophy [21]. In the ideal scenario, the relative nonlinear parameter is independent of the excitation amplitudes, observable from its definition (in terms of the amplitude to the square of the amplitude ratio). Therefore, it is only related to the nonlinear system itself [4]. Due to these appealing characters, the TNP has been used for decades to characterize the second harmonic responses.

It is important to note that the concept of the TNP originates from the bulk wave case under the assumption that the amplitude of the fundamental waves remains unchanged during the wave propagation process [4]. This may be irrelevant if the TNP is used to track the structural or material changes when the distance between the actuator and sensor is fixed. When it comes to the characterization of the cumulative effects of the second harmonic Lamb waves with the changing distances, however, this assumption, as well as its consequences in the SHM applications becomes questionable. In one-dimensional wave propagation cases (one-dimensional (1D) cases), the plane wave theory can be roughly applied, as used in many theoretical and numerical investigations [12,15,16]. In these cases, the gradual increase in the amplitudes of the MNP-induced second harmonic Lamb waves results in an expected gradual increase in the TNP, since the propagating Lamb waves are nondiverging. In any practical SHM applications, however, Lamb waves are usually generated within a confined region by actuators of finite size such as piezoelectric wafers. Waves then propagate in a two-dimensional (2D) pattern cases, becoming nonplanar and divergent so that their amplitudes decrease with the propagation distance. Therefore, the independence of the TNP on the amplitude of fundamental waves, established in 1D cases, may be compromised. As a result, the legitimacy of using the TNP to characterize the MNP needs to be re-examined. Moreover, due to the existence of some inevitable nonlinear sources in the system, particularly the AN, it is also crucial to examine whether the specific noncumulative nonlinear source can be properly characterized by the TNP.

In this paper, the distributed MNP and the localized nonlinear sources like the AN in both 1D and 2D cases are characterized. Analyses first highlight the deficiencies of the TNP in 2D nonplanar applications. Then, a refined nonlinear parameter (RNP) is proposed based on the wave propagation characteristics for a better characterization of both the cumulative MNP-induced and the noncumulative AN-induced nonlinear Lamb waves. Finite element (FE) simulations are performed with the AN and the MNP being tactically separated in the models. Upon extracting the amplitudes of the time-domain signals using a peak tracking method, the TNP and the refined nonlinear parameter are compared in terms of their ability to characterize the second harmonic responses, which demonstrates the superiority of the proposed nonlinear parameter. Finally, a previously developed model [18] is used to guide the design of two experimental configurations, an optimized and an un-optimized. These purpose-driven experimental configurations are conceived to adjust the dominance level of the linear and the AN-induced Lamb waves, propagating from M to N through a direct path, one has

$$\frac{A_{NL}(\omega)}{A_{L}(\omega)} = \frac{A_{NL}(2\omega)}{A_{M}(2\omega)}$$

(2)

where $A(\omega)$ and $A(2\omega)$ are the amplitudes of Lamb waves at the fundamental and the double frequencies, respectively. Subscripts M and N denote the two positions under consideration, and subscripts L and NL represent the linear and nonlinear wave components, respectively. Using TNP, the slope of the TNP-$\alpha$ curve writes

$$S_{L} = \frac{A_{NL}(2\omega)}{A_{L}(\omega)} \frac{1}{\Delta x} = \frac{A_{NL}(2\omega)}{A_{L}(\omega)} \frac{1}{\Delta x}$$

(3)

Fig. 1 Sketches of the propagating patterns of the wave components induced by different nonlinear sources: (a) the AN and (b) the MNP (the amplitude of the second harmonic waves should be much smaller than that of the fundamental waves).

2 Theoretical Analyses

Two important nonlinear sources, the distributed MNP and a representative localized AN, present in a typical NL-SHM system actuated by the piezoelectric wafers, are considered. Specifically, problems associated with the TNP for characterizing the second harmonic Lamb waves in the nonplanar 2D cases, are scrutinized. Based on the physical understandings revealed by the theoretical analyses, a refined nonlinear parameter is proposed, aiming at a better characterization and differentiation of the cumulative MNP-induced and the noncumulative AN-induced second harmonic Lamb waves in the system.

2.1 Characterization of the Adhesive Nonlinearity-Induced Second Harmonic Lamb Waves With Traditional Nonlinear Parameter. As shown in Ref. [18], the influence of the AN at the actuator-plate interaction is more significant than that at the sensing part in terms of the second harmonic responses. Therefore, analyses will mainly focus on the adhesive layer over the actuation area. In an ideal 1D case, the AN-induced second harmonic Lamb waves propagate independently with the linear wave components. In the absence of the MNP and the wave attenuation, the amplitudes of both the linear and nonlinear Lamb waves remain unchanged during the propagation as sketched in Fig. 1(a). Using the TNP, the slope of the TNP-$\alpha$ curve should then be zero, with $\alpha$ being the propagation distance from the actuator. In addition, this zero-slope feature should be independent of the material nonlinear properties of the adhesive layers.

In practical 2D cases, the attenuation of Lamb waves can be mainly attributed to two factors: wave divergence and material damping. The present work mainly focuses on the S$_{0}$ mode Lamb wave propagation in an aluminum plate in the low-frequency range, within which the wave attenuation due to the material damping is much weaker than that due to the wave beam divergence. Therefore, the former, which is frequency-dependent, in principle, is considered to be negligible. Of course, this will not be applicable to other more dissipative media such as composite plates. As far as the wave divergence is concerned, it has been shown that the amplitude of Lamb waves decreases as a function of the square root of the propagating distance, but independent of the frequency [26]. Based on this and considering both the linear and the AN-induced Lamb waves, propagating from M to N through a direct path, one has

$$A_{NL}(2\omega) / A_{L}(\omega) = \frac{A_{NL}(2\omega)}{A_{M}(2\omega)}$$

where $A(\omega)$ and $A(2\omega)$ are the amplitudes of Lamb waves at the fundamental and the double frequencies, respectively. Subscripts M and N denote the two positions under consideration, and subscripts L and NL represent the linear and nonlinear wave components, respectively. Using TNP, the slope of the TNP-$\alpha$ curve writes

$$S_{L} = \frac{A_{NL}(2\omega)}{A_{L}(\omega)} \frac{1}{\Delta x} = \frac{A_{NL}(2\omega)}{A_{L}(\omega)} \frac{1}{\Delta x}$$

(3)

Fig. 1 Sketches of the propagating patterns of the wave components induced by different nonlinear sources: (a) the AN and (b) the MNP (the amplitude of the second harmonic waves should be much smaller than that of the fundamental waves).
where \( \Delta x \) denotes the distance between the two points M and N. Equation (3) shows a product of two terms in which the first one only involves the wave amplitudes, which is always positive. The same applies to the second term, since \( A^N_M(\omega) < A^N_M(\omega) \) due to the 2D wave divergence. As a result, SL is always positive, giving an upward variation trend of the TNP with the propagation distance. This shows a false-positive cumulative effect in the nonplanar 2D scenario, which should not exist in the absence of the MNP.

### 2.2 Characterization of Material Nonlinearity of a Plate-Induced Second Harmonic Lamb Waves With Traditional Nonlinear Parameter

Considering the damage-related MNP in the same low-frequency range, we focus on the cumulative MNP-induced \( S_N \) mode Lamb waves. In 1D cases, the amplitudes of the linear wave components remain almost constant during the wave propagating while the nonlinear wave amplitudes gradually increase due to the distributed material nonlinearity, as illustrated in Fig. 1(b). Without loss of generality, the amplitude of the MNP-induced cumulative second harmonic Lamb waves at a position N can be expressed as

\[
A^N(2\omega) = A^N_M(2\omega) + k_{\text{material}}(A^M_M(\omega))^2 \Delta x
\]

Equation (5) shows a constant slope, which is only related to the material nonlinear elastic properties at a specific excitation frequency. As \( A^N_M(\omega) = A^L_M(\omega) \) in the present 1D case, the SL in the TNP-x figure can be expressed as

\[
\text{SL} = \frac{A^N(2\omega)}{A^L(2\omega)} = \frac{(A^N_M(\omega))^2 - (A^L_M(\omega))^2}{(A^L_M(\omega))^2} \Delta x
\]

\[
= \frac{A^N_M(2\omega) + k_{\text{material}}(A^M_M(\omega))^2 \Delta x}{(A^L_M(\omega))^2} - \frac{A^N_M(2\omega)}{(A^L_M(\omega))^2} = k_{\text{material}}\Delta x
\]

Equation (5) shows a constant slope, which is only related to the material nonlinear elastic properties at a specific excitation frequency, in agreement with previous studies [15,16].

In a nonplanar 2D case, when the linear and the MNP-induced Lamb waves propagate from M to N through a direct path, the amplitude of the second harmonic Lamb waves at N can be decomposed into two parts as

\[
A^N_M(2\omega) = A^N_M(1\omega) + \left[ \int_M^N k_{\text{material}}(A^L_M(\omega))^2 dx \right] f(x)
\]

where the first part \( A^N_M(1\omega) \) denotes the nonlinear wave amplitude without the cumulative effect. Analog to Eq. (2), one has \( (A^N_M(\omega)/A^L_M(\omega)) = (A^N_M(2\omega)/A^M_M(2\omega)) \). The second part is related to the cumulative effect due to the MNP, in which \( f(x) \) describes the attenuation of the cumulative Lamb wave components due to the wave divergence. Using TNP, the SL of the TNP-x curve can be expressed as

\[
\text{SL} = \frac{A^N_M(2\omega)}{(A^L_M(\omega))^2} - \frac{A^N_M(2\omega)}{(A^L_M(\omega))^2} \Delta x
\]

\[
= \frac{A^N_M(2\omega) + \left[ \int_M^N k_{\text{material}}(A^L_M(\omega))^2 dx \right] f(x)}{(A^L_M(\omega))^2} - \frac{A^N_M(2\omega)}{(A^L_M(\omega))^2} = \frac{1}{k_{\text{material}}(A^L_M(\omega))^2} \Delta x
\]

The above expression shows that the SL slope is no longer a constant. Instead, it is affected by both the attenuation of the Lamb waves due to the wave divergence effect (the first term and the second term) and the material status (the second term). Particularly, the slope tends to be larger when the attenuation of Lamb waves becomes severer.

The above analyses surmise that the commonly used TNP may lead to an imprecise prediction of the SL for the distributed nonlinearity on one hand and a false characterization of the localized nonlinearity on the other hand, due to the wave divergence in 2D cases. All in all, this may jeopardize the damage diagnosis in the cumulative effect-based SHM methods. Numerical evidences will be given in Sec. 3.3 to ascertain the previously mentioned theoretically predicted SL variations.

### 2.3 A Refined Nonlinear Parameter

To tackle the problem, a so-called RNP is proposed based on the wave propagating characteristics as

\[
\text{RNP} = \frac{A^N_M(2\omega)}{A^L_M(2\omega)}
\]

where \( A^N_M(2\omega) \) is the amplitude of the second harmonic Lamb waves with the excitation at the fundamental frequency \( \omega \), \( A^L_M(2\omega) \) is the amplitude of the linear Lamb waves which are excited at the double frequency \( 2\omega \) and propagate independently in the plate. Due to the wave divergence, the amplitude of the propagating second harmonic waves follows the same variation pattern as their linear counterparts at the double frequency so that the proposed RNP allows the compensation for the wave attenuation in 2D cases. Therefore, the physical nature of the proposed RNP is a compensated second harmonic wave descriptor, to be used as an alternative to the conventional \( \beta' \), which hopefully is more conducive to SHM applications.

As the AN-induced second harmonic Lamb waves propagate independently in the plate, their attenuation pattern should be similar to that of the linear Lamb waves at the double frequency. Therefore, to characterize the AN according to Eq. (8), the RNP should be independent on the propagating distance and the slope of the RNP-x curve (\( \text{SL}_\text{refined} \)) should theoretically be zero. Compared with the SL obtained using the TNP, the nondamage related nonlinear source characterized with the RNP will no longer induce the false-positive cumulative effect.

As to the MNP, the SL\(_\text{refined} \) can be expressed as

\[
\text{SL}_\text{refined} = \frac{A^N_M(2\omega)}{A^L_M(2\omega)} - \frac{A^N_M(2\omega)}{A^L_M(2\omega)} \Delta x
\]

\[
= \frac{A^N_M(2\omega) + \left[ \int_M^N k_{\text{material}}(A^L_M(\omega))^2 dx \right] f(x)}{A^L_M(2\omega)} - \frac{A^N_M(2\omega)}{A^L_M(2\omega)} = \frac{1}{k_{\text{material}}(A^L_M(\omega))^2} \Delta x
\]

Equation (9) indicates a positive slope so that the cumulative effect can still be characterized with the RNP. Moreover, compared with Eq. (7), the term which is merely related to the wave attenuation is eliminated (the first term in Eq. (7)). In other words, a better compensation for the wave attenuation in 2D cases can be achieved with the proposed RNP.

To sum up, in practical 2D cases with the presence of both the AN and the MNP in an SHM system, the proposed RNP is expected to pinpoint the cumulative effect from the measurements, which can only be induced by MNP. Meanwhile, the
cumulative MNP-induced second harmonic Lamb waves can also be better characterized with the RNP through the proper compensation for the Lamb wave attenuation. These advantages can offer great benefit for the further SHM applications. The previously mentioned analyses will be ascertainment and validated in Secs. 3 and 4 through numerical and experimental analyses.

3 Finite Element Validations

In this section, FE validations are carried out. To start with, the noncumulative feature associated with the AN and the cumulative effect corresponding to the MNP are validated with 2D FE models under the plane wave assumption, laying out the benchmark for further discussions. Then, three-dimensional (3D) FE models are established to simulate typical nonplanar 2D wave propagating cases, along with a dedicated peak tracking method to extract the amplitude of the responses. Through analyses, the deficiencies of the TNP are addressed while the effectiveness of the proposed RNP are demonstrated.

3.1 Mechanism Validations

As the foundation of the subsequent analyses, the propagating features of the AN-induced and the MNP-induced second harmonic Lamb waves need to be validated within our analytical framework. A 2D FE model is established to investigate the 1D propagating Lamb waves using ABAQUS, as illustrated in Fig. 2. Two piezoelectric actuators (20 mm wide and 0.5 mm thick) are symmetrically bonded on a 2 mm-thick aluminum plate to generate $S_0$ mode Lamb waves. As to the wave sensing, five piezoelectric sensors (8 mm wide and 0.5 mm thick) are bonded on the plate from 100 mm to 300 mm away from the actuators, with a separation distance of 50 mm between each pair. The thicknesses of the adhesive layers are all set to 0.03 mm. The nonlinear material properties of both the adhesive layer and the plate can be included or excluded as needed. The nonlinear material behaviors are programmed in the UMAT module. Relevant parameters of the piezoelectric transducers, adhesive layers and plate used in the FE models are tabulated in Table 1. The excitation is a 70 kHz five-cycle tone burst signal. The nonlinear responses are extracted in the time domain with the superposition method in which two signals with inverse phases are separately used as the excitations and their corresponding responses are superposed [18]. The use of five-cycle tone burst signal is to ensure a good temporal resolution, necessary for analyses. Since we use the $S_0$ Lamb waves in the low-frequency range, wave dispersion should be rather weak. Meanwhile, as the second harmonic Lamb waves are extracted using the superposition method in the time domain, only the wave amplitude matters instead of the cycles, which has more impact on the generation efficiency of the second harmonics.

First, only the AN is included in the model. As the amplitudes of the linear responses are much larger than their nonlinear counterparts, the overall responses of the sensors can be taken as the linear responses, as shown in Fig. 3(a). It can be seen that, apart from an offset in the time domain, the wave patterns as well as the amplitudes of the linear responses remain almost unchanged (the dotted line) during the wave propagation, as expected. A similar observation also applies to the nonlinear responses induced by the AN, as illustrated in Fig. 3(b). This shows the noncumulative nature of the AN-induced nonlinear waves, as predicted in the theoretical analyses. Then, only the MNP is introduced to the system and the results are shown in Fig. 3(c). The amplitudes of the MNP-induced nonlinear Lamb waves increase gradually as indicated by the dotted line, demonstrating the typical cumulative effect within the chosen propagation region. Therefore, both the cumulative effect of the MNP and the noncumulative effect of the AN are ascertained, which is consistent with the theoretical analyses.

3.2 Three-Dimensional Finite Element Model and the Peak-Tracking Method for Amplitude Extraction

To simulate the more practical 2D wave propagation cases, a 3D FE model is built with symmetric square piezoelectric actuators bonded on an aluminum plate as illustrated in Fig. 4. Only a quarter of the system is required with symmetric boundary conditions at the left and the bottom ends. Other details regarding the piezoelectric wafers (for both actuation and sensing) remain the same as those used in the previously mentioned 1D case. In addition, the infinite elements are applied at the top edge of the plate to reduce the problem size and eliminate the edge reflections at the same time [27]. Again, the AN and the MNP are separately introduced into the model as needed. Excitation frequencies at 70 kHz or 140 kHz with five-cycle tone burst signals are used, respectively.

In the low-frequency range of our specific interest, reflections from the bonded sensors may be mixed with the incident Lamb waves since the wavelength and the distance between the sensors are comparable. In this case, traditional methods like the wavelet transform cannot precisely extract the amplitude of incident $S_0$ mode wave packages in the time-domain responses. Using the case of 140 kHz excitation as an example, the voltage responses of five sensors are shown in Fig. 5(a). The complex Morlet wavelet transform would give the amplitudes of Lamb wave signals as shown in Fig. 5(b) [28]. It can be seen that the amplitude at 250 mm is even larger than that at 200 mm. This contradicts with the expected monotonically decreasing trend of the wave amplitudes associated with the divergence of Lamb waves. Nevertheless, as it takes some time for the reflections to be mixed with the incident waves, the very first parts of the $S_0$ mode Lamb wave signals in the time-domain tend to be less polluted. Based on this, a

![Fig. 2 The 2D FE model](image-url)
peak tracking method is proposed to extract the amplitudes of Lamb wave responses by using the two amplitude quantities of the first obvious peak in the signal, marked by A and B in the example shown in Fig. 5(c). Through the normalization to the amplitude of the sensor output at 100 mm away from the actuator, attenuation curves of Lamb waves at 140 kHz are obtained and shown in Fig. 5(d). A relatively close agreement can be observed between the results obtained by peak tracking A and B. The average of the two is then used, showing a more physical and consistent decreasing trend of the wave amplitude as opposed to the one.

Fig. 3 Results of 1D wave propagation cases: (a) overall (linear) responses, (b) nonlinear responses induced by the AN, and (c) nonlinear responses induced by the MNP

Fig. 4 The 3D FE model
given by Morlet wavelet transform. It should be noted that the transient vibration and ringing effect may in principle exist [29]. In the present case, however, the captured signals show highly consistent and clear wave patterns and are well synchronized with the excitation frequency around the first two peaks. Therefore, the aforementioned phenomena, if any, should be negligible. In the remaining part of the paper, this peak tracking method will be used to extract the wave amplitudes of various components as needed in the analyses.

3.3 Characterization of the Second Harmonic Lamb Waves With Traditional Nonlinear Parameter and Refined Nonlinear Parameter in Two-Dimensional Cases. First, only the AN is introduced to the previously mentioned 3D FE model. The excitation frequency is set to 70 kHz and the linear voltage responses of the sensors at 100, 150, 200, 250, and 300 mm are captured and shown in Fig. 6(a). The aforementioned peak tracking method is used, giving the comparisons shown in Fig. 6(b) between the wave attenuation at 70 kHz and 140 kHz. The similar decreasing trend for both cases agrees with the theoretical analyses in Sec. 2.1.

The superposition method is then applied to extract the AN-induced second harmonic responses in the time domain. First, according the definition of the TNP, the AN-induced second harmonic voltage responses are compensated with the square of the linear wave amplitudes at 70 kHz in Fig. 6(b), resulting in the nonlinear responses of the sensors in the time domain as shown in Fig. 7(a). It is relevant to note that the values of the amplitudes have no physical meanings and only the trend of these nonlinear responses at different sensing positions is of interest. After further extracting the amplitudes of the responses and carrying out the normalization to the amplitude of the first sensor output, the normalized TNP-x figure is shown in Fig. 7(b). It can be seen that a false-positive cumulative effect indeed occurs as an artifact by the TNP in the present case where only the AN is considered. This is also consistent with the prediction of previous theoretical analyses.

To use the RNP, the Lamb waves are excited at 140 kHz in a separate run. Then, the AN-induced second harmonic responses are compensated with the linear wave amplitudes at 140 kHz, giving results shown in Fig. 7(c). Upon the further amplitude extraction and the normalization to the amplitude of the closest sensor signal, the normalized RNP-x figure is obtained and plotted in Fig. 7(d). The slope is very close to zero, truthfully indicating the non-cumulative nature of the AN.

Following the same process, only the MNP is introduced to the system. Both the TNP and the RNP are used to characterize the second harmonic responses, with the results shown in Fig. 8. The nonlinear responses after compensations are shown in Figs. 8(a) and 8(c), regarding to the TNP and RNP, respectively. By further extracting the wave amplitudes and carried out the normalization process, the normalized TNP-x figure and the normalized RNP-x figure are shown in Figs. 8(b) and 8(d), respectively. Compared with the results using TNP (Fig. 8(b)), a better linear variation is
obtained with the proposed RNP (Fig. 8(d)). This demonstrates that the wave attenuation is effectively compensated for when the RNP is used to characterize the MNP-induced second harmonic Lamb wave responses. This also agrees with the previous theoretical analyses.

To conclude, all the FE evidences support the previous theoretical analyses and demonstrate that the proposed RNP is more suitable for characterizing the nonlinear sources of different natures. First, the proposed RNP can effectively avoid the false-positive cumulative effect in the presence of the localized nonlinear

Fig. 6 (a) Linear responses at 70 kHz from the 3D FE model and (b) comparison of the wave attenuation between 70 kHz and 140 kHz cases

Fig. 7 (a) The AN-induced $S_0$ waves compensated with the square of the linear wave amplitudes at 70 kHz according to the definition of the TNP, (b) the normalized TNP-x figure for the AN case, (c) the AN-induced $S_0$ waves compensated with the linear wave amplitudes at 140 kHz according to the definition of the RNP, and (d) the normalized RNP-x figure for the AN case.
sources such as the AN. Second, the cumulative effect due to the MNP can be better characterized with a fairly good linearity due to the appropriate compensation for the wave attenuation in 2D cases. Both outcomes should be beneficial to damage diagnosis in the cumulative effect-based SHM methods.

4 Experimental Validations

The theoretically and numerically predicted advantages of the RNP are also experimentally validated. To this end, two typical system configurations are tactically conceived. Experiments are then carried out.

4.1 Design of the System Configurations. In practical applications, both the AN and the MNP are inevitably present in a piezoelectric wafer-actuated SHM system. To experimentally validate the ability of the proposed RNP in characterizing and differentiating these two types of nonlinearities, two systems should be designed: the first one is referred to as the optimized system with dominant MNP and the other one the un-optimized system with more significant AN. The relative dominance level of the MNP can be achieved through manipulating the AN level in the system, which is materialized with the help of a previously developed model [18]. The level of the AN has been shown to be closely dependent on the size of the actuator as well as the excitation frequency due to the frequency tuning characteristics of the AN-induced $S_0$ mode Lamb waves [18]. The previously developed nonlinear shear-lag model allows accurate tuning and optimization of the actuator size for a given excitation frequency. In the present case, the optimized system corresponds to the valley of the frequency tuning curves, while the un-optimized one should be close to its peak. In the former, the AN is minimum, thus allowing the dominance of the MNP, while in the latter, the maximized AN exceeds and dominates the MNP. Without detailing the procedure, we design the configurations of the piezoelectric actuators for the fundamental frequency at 70 kHz, giving the following two system configurations. The widths of the actuators are 32 mm and 8 mm for the optimized and un-optimized systems, respectively.

Before experiments, a 2D FE model is used again to ascertain the efficacy of the system design process. For this specific purpose, the piezoelectric actuators are only bonded on one side of the plate, to be in-line with the subsequent experiments. Configurations and material properties of the adhesive layers, the plates and piezoelectric sensors are identical to those used in the previous 2D FE model. The relative influences of the AN and the MNP are compared in terms of their corresponding nonlinear responses captured by the sensor located 200 mm away from the actuator, as shown in Fig. 9. It can be seen that the AN is indeed dominant in the un-optimized system, as evidenced by its overwhelming amplitude as compared to that of the MNP (Fig. 9(a)). In the optimized system, however, the level of the AN is significantly weakened, leading to a clear dominance of the MNP at the early part of the signals ($S_0$ mode Lamb wave responses).

4.2 Experimental Setup. In the experiments, six square piezoelectric wafers are bonded on a 2 mm-thick aluminum plate,
The piezoelectric wafers are bonded by applying the same pressure to ensure the bonding conditions to be as consistent as possible. The marks and the geometric locations of the piezoelectric transducers are shown in Fig. 10(a). Actuator A1 has a width of 32 mm and all the others are 8 mm wide. The optimized system contains actuator A1 and sensors S1 to S4, in which the influence of the MNP should be dominant. The combination of actuator A2 and sensors S1 to S4 forms the un-optimized system in which the AN is expected to be more significant. The whole experimental setup is shown in Fig. 10(b) with other details on the measuring system given in our previous work [18]. The excitation voltage is set to 200 V. For each system configuration, the experiments are carried out following a two-step procedure:

1. Excite the system at 140 kHz and capture the responses; use the peak tracking methods to extract the wave amplitudes;
2. Excite the system at 70 kHz twice with inverse phases and capture the responses; extract the nonlinear responses with the superposition method and calculate the normalized RNP-x figure.

4.3 Results. The optimized system is first tested. Responses at 140 kHz are first captured, as shown in Fig. 11(a). Using the peak tracking method, the corresponding attenuation curves are obtained and shown in Fig. 11(b). Then, the responses at 70 kHz with the inverse excitations are superposed to obtain the nonlinear responses. Two hundred measurements are taken and averaged to minimize the influence of the background noise. Through the compensation with the linear wave amplitudes at 140 kHz according to the definition of the RNP, the nonlinear responses of the sensors at different positions are shown in Fig. 11(c). After extracting the amplitudes with the peak tracking method and carrying out the normalization to the first amplitude, the normalized RNP-x figure is obtained and shown in Fig. 11(d). A clear cumulative effect can be observed from the results. This is consistent with our expectations that the influence of the MNP dominates in the system.

In the un-optimized system, following the same process, the linear responses and the attenuation curves of Lamb waves at 140 kHz are shown in Figs. 12(a) and 12(b), respectively. Similarly, the nonlinear responses and the normalized RNP-x curve...
can be finally obtained, as illustrated in Figs. 12(c) and 12(d). In this case, as the AN is dominant in the system, Fig. 12(d) shows no cumulative effect, which is also in agreement with our expectations.

To sum up, experimental evidences demonstrate that the proposed RNP can effectively characterize both AN-induced and MNP-induced second harmonic responses in terms of their cumulative characteristics in 2D cases, which should offer great benefit for further SHM applications.

5 Conclusions

In this paper, the SHM-promising second harmonic Lamb waves in nonplanar 2D wave propagating cases are investigated in the perspective of their characterizations and quantifications. Under the assumption of the weak material damping in metallic plates, both the primary and the second harmonic undergo attenuations, dominated by the wave divergence. Theoretical analyses first show the deficiency of the widely used traditional nonlinear parameter to characterize the cumulative features of the second harmonic Lamb waves with changing measuring distances. As an alternative, a so-called refined nonlinear parameter is proposed. The new parameter considers the wave propagating characteristics in a practical 2D configuration, aiming at a better characterization and differentiation of the localized and distributed nonlinear sources. A peak tracking method is used for wave amplitude extractions. The efficacy of the proposed RNP is then validated through the FE simulations in terms of characterizing both the cumulative effect associated with the MNP and the noncumulative effect corresponding to the AN. Advantages of the proposed RNP, predicted by both theoretical and numerical analyses, are then experimentally validated using two tactically conceived systems with the help of a previously developed model. Through testing the two systems, the feasibility as well as the advantages of the proposed RNP is ascertained.

It is shown that the wave divergence of the Lamb waves in a nonplanar 2D configuration, along with the presence of the localized nonlinear sources, excites a vital influence on the characterization of the cumulative effect of the second harmonic Lamb waves for SHM. This challenges the legitimacy and the accuracy of the TNP for the nonlinear wave characterizations and further for the cumulative effect-based SHM methods. By contrast, the proposed RNP is shown to entail a better characterization of the nonlinear sources of different natures in the system, as evidenced by both FE and experimental results. The advantages of the proposed RNP mainly lie in two aspects. First, using the RNP, the false-positive cumulative effect due to AN can be avoided, which is particularly appealing for further SHM applications; Second, the cumulative effect induced by the MNP can be better characterized with a fairly good linearity due to the proper compensation for the attenuation of the propagating Lamb waves.

The slope of the normalized RNP-x curve, derived from the proposed RNP, can be further extracted as an indicator for the detection of the microstructural changes in structures, since any changes in the slope can only be attributed to the material degradation of the plate other than other localized nonlinear sources.
such as the adhesive nonlinearity. Further investigations on the relevant SHM methods based on the newly proposed nonlinear parameter should be explored in our future work.

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