Numerical analysis of multi-layer composite plates with internal delamination

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Abstract

A three-dimensional finite element model for the delaminated fiber reinforced composites is proposed to analyze the dynamics of multi-layer composite plates with internal delamination. Virtual elements are adopted in the region of delamination to prevent element penetration. Natural frequency, modal displacement and modal strain are analyzed for samples with different dimensions of delamination. Numerical results show a good agreement with the available experimental data and an enhancement of the accuracy of the results when the proposed model is adopted. The results of this study are useful for detecting delamination in multi-layer composite materials.

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Keywords: Multi-layer composites; Finite element model; Virtual elements; Internal delamination; Modal parameters; Numerical analysis

1. Introduction

The use of composite materials in space vehicles and various machine components has increased considerably over the past decades. Under repeated or impact loads these materials are subjected to various forms of damage, mostly delaminations and cracks [1–4]. Such damage becomes an obstacle to the more extensive usage of composite materials. Therefore, the monitoring of internal or hidden damage in composite material is critical in engineering practice [5]. The use of vibration-based techniques as nondestructive testing methods for damage monitoring of laminated composite is a field attracting the interest of many researchers [6–12].

The effective damage monitoring for this kind of material or structure depends largely on the accurate prediction or estimation of mechanical or dynamic behaviors of both intact and damaged composite materials [13]. As it is difficult to obtain accurate analytical solution for multi-layer material problem theoretically, the computational approach, e.g., finite element method, plays an important role in the implementation of damage detection for laminated composites. There are contributions on both numerical and experimental investigations into the behavior of delaminated multi-layer composites [14–19].

After Sankar [20] had modeled a delaminated beam as two sublaminates by offset beam finite elements, Ricards [21] developed a model of finite superelements for sandwich composite beam and plate without delamination, each layer being considered as a simple Timoshenko beam. Later, Gabelrab [3] discussed the modal variation of delaminated beam for different boundary conditions and Ousset and Roudolff [4] analyzed the delaminated multi-layer composite plate based on Mindlin-Reissner plate model. Zak et al. [22] and Ousset and Roudolff [4] developed models of finite elements for beams and plates with boundary delamination. Among most of the publications the prediction of material mechanical or dynamic behaviors is based on the classical laminated plate theory [23].

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the transverse shear deformation effect, the classical layerwise theory based on the straight normal assumption of elastic thin plate cannot provide accurate results for moderately thick laminated plates, for which the in-plane elastic modulus is much higher than the transverse shear modulus [24]. Besides, the Poisson’s effect is significant for angle-ply laminated plates. Therefore, three-dimensional layerwise theory must be adopted in order to obtain an accurate prediction of dynamic response for multi-layer composite plates. In addition, potential dangers are often induced by hidden or internal delaminations in the in-service laminated composites. However, there is a lack of both numerical and experimental investigations on internal delaminations with different geometries in laminated composites.

In this paper, a three-dimensional finite element model for multi-layer composites with internal delaminations is established, and the fiber orientation of individual lamina as well as the transverse shear effect are taken into account. Numerical calculation is carried out for different plates. Natural frequencies, modal displacements and strains of the intact and damaged multi-layer composite plates are subsequently analyzed for various samples.

2. Three-dimensional elastic theory for laminated composite plates

2.1. Geometric equation

By introducing a Cartesian global coordinate system $x$, $y$, $z$ to a rectangular plate with uniform thickness as shown in Fig. 1, the displacements of a point $(x, y, z)$ in the plate can be expressed as

$$\{u\} = (u_1, u_2, u_3)^T = (u(x, y, z), v(x, y, z), w(x, y, z))^T$$ (1)

where $u$, $v$ and $w$ represent the displacements along $x$, $y$ and $z$ axes, respectively.

The linear strain–displacement relationships are

$$e_{ij} = \frac{1}{2} (u_{ij} + u_{ji}) \quad (i, j = 1, 2, 3)$$ (2)

2.2. Physical equation

Assume that the individual lamina is orthotropic with fiber orientation along $x_1$ axis of the Cartesian coordinate system $x_1$, $x_2$, $x_3$ (local material coordinates) as shown in Fig. 1, the constitutive equation for the lamina is

$$\{\sigma^\ast\} = [C]\{\varepsilon^\ast\}$$ (3)

where

$$\{\sigma^\ast\} = (\sigma_{11}^\ast, \sigma_{22}^\ast, \sigma_{33}^\ast, \sigma_{12}^\ast, \sigma_{13}^\ast, \sigma_{23}^\ast)^T$$ and

$$\{\varepsilon^\ast\} = (\varepsilon_{11}^\ast, \varepsilon_{22}^\ast, \varepsilon_{33}^\ast, \varepsilon_{12}^\ast, \varepsilon_{13}^\ast, \varepsilon_{23}^\ast)^T$$

are vectors of stress and strain along the main directions in local coordinate system, respectively. $[C]$ is the elastic constant matrix of the material expressed as

$$[C]^{-1} = \begin{bmatrix}
\frac{1}{E_{11}} & \frac{-v_{12}}{E_{12}} & \frac{-v_{13}}{E_{13}} & 0 & 0 & 0 \\
\frac{-v_{12}}{E_{12}} & \frac{1}{E_{22}} & \frac{-v_{23}}{E_{23}} & 0 & 0 & 0 \\
\frac{-v_{13}}{E_{13}} & \frac{-v_{23}}{E_{23}} & \frac{1}{E_{33}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{31}}
\end{bmatrix}$$ (4)

$E_1$, $E_2$, $E_3$, $G_{12}$, $G_{13}$, $G_{23}$, $v_{12}$, $v_{13}$ and $v_{23}$ are the orthotropic elastic constants of the lamina.

The stresses in the global coordinate system can be obtained as follows:

$$\{\sigma\} = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xz}, \sigma_{xy}, \sigma_{yz})^T = [A]\{\sigma^\ast\}$$ (5)

where $[A]$ is the matrix of directional cosines relevant to coordinate transformation from local material coordinates to the global ones, i.e.

$$[A] = \begin{bmatrix}
\cos^2 \theta & \sin \theta \cos \theta & 0 & 0 & 0 & -2 \sin \theta \cos \theta \\
\sin \theta \cos \theta & \sin^2 \theta & 0 & 0 & 0 & 2 \sin \theta \cos \theta \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \theta & \sin \theta & 0 \\
0 & 0 & 0 & -\sin \theta & \cos \theta & 0 \\
\sin \theta \cos \theta & -\sin \theta \cos \theta & 0 & 0 & 0 & \cos^2 \theta - \sin^2 \theta
\end{bmatrix}$$ (6)

Therefore, the constitutive equation in the global coordinate system can be expressed as

$$\{\sigma\} = [A][C][A]^{-1}\{\varepsilon\}$$ (7)
where

\[ \varepsilon = (e_{xx}, e_{yy}, e_{zz}, e_{xy}, e_{xz}, e_{yz})^T \]

is the strain vector in the global coordinate system \( x, y, z \).

### 3. Finite element modeling

#### 3.1. Element model description

The finite element used for multi-layer plate dynamic behavior analysis is a kind of eight-node rectangular thin plate element as shown in Fig. 2. For each node, there are three degrees of freedom, i.e., translations along \( x, y \) and \( z \) axes. The element thickness is assigned to be equal to that of the corresponding individual lamina, which may not be the same for all the elements. The element coordinate system is arranged such that the first axis is coincident with the fiber direction. All physical parameters throughout an element are assumed to be the same.

#### 3.2. Expressions for displacement and strain

For an eight-node finite element with three degrees of freedom per node, the displacement field over an element is given by

\[ \{u\} = \sum_{i=1}^{8} [N_i]\{\delta_i\} \tag{8} \]

where

\[ \{\delta_i\} = (u_i, v_i, w_i)^T \]

is the displacement vector at node \( i \) and

\[ [N_i] = N_i[J_i] \]

where \([J_i]\) is a three-order unit matrix and \( N_i \) the shape function for node \( i \) [25]. Then the strain of each element can be expressed in terms of displacement in the global coordinate system as

\[ \{\varepsilon\} = \sum_{i=1}^{8} [B_i]\{\delta_i\} \tag{9} \]

where

\[ [B_i] = [A][N_i] \tag{10} \]

and

\[ [A] = \begin{bmatrix} \frac{\partial \varepsilon_{xx}}{\partial x} & 0 & 0 \\ 0 & \frac{\partial \varepsilon_{yy}}{\partial y} & 0 \\ 0 & 0 & \frac{\partial \varepsilon_{zz}}{\partial z} \end{bmatrix} \tag{11} \]

Thus, for a given element, the strain can be expressed in terms of nodal displacements as

\[ \{\varepsilon\} = [B]\{\delta\} \tag{12} \]

where

\[ [B] = [[B_1], [B_2], \ldots, [B_8]] \]

and

\[ \{\delta\} = ([\delta_1]^T, \ldots, [\delta_8]^T)^T. \]

#### 3.3. Stress–strain relationship and equation of motion

According to Eqs. (7) and (12), the stresses of an element in the global coordinate system can be expressed by the nodal displacements as

\[ \{\sigma'\} = [A][C][A]^{-1}[B]\{\delta\} \tag{13} \]

Thus, the element stiffness matrix may be written as

\[ [K'] = \int_{V_e} [B]^T[A][C][A]^{-1}[B] \, dV \tag{14} \]

Then, the strain energy of the \( k \)th element is

\[ U_k' = \frac{1}{2} \int_{V_e} \{\varepsilon'\}^T\{\sigma'\} \, dV = \frac{1}{2} \{\delta_k\}^T[K'_k]\{\delta_k\} \tag{15} \]

where \( \{\delta_k\} \) represents the displacement vector of the \( k \)th element. The total strain energy for a composite plate consisting of \( N \) elements can be expressed as

\[ U = \sum_{k=1}^{N} U_k' = \frac{1}{2} \sum_{k=1}^{N} \{\delta_k\}^T[K'_k]\{\delta_k\} \tag{16} \]

Therefore, after assembly of nodal displacements of all elements, the total strain energy of a multi-layer composite plate can be represented as

\[ U = \frac{1}{2}\{\delta\}^T[K]\{\delta\} \tag{17} \]

where \( \{\delta\} \) and \( [K] \) are the global nodal displacement vector and stiffness matrix, respectively.
Similarly, if the composite plate experiences a harmonic motion with angular frequency $\omega$, the kinetic energy of the composite plate will be

$$T = \frac{1}{2} \omega^2 \{\delta\}^T [M] \{\delta\}$$

(18)

where $[M]$ is the global mass matrix. Then, using the Lagrange’s principle the equation of motion for free vibration of the composite plate is reduced to the eigenvalue problem of

$$([K] - \omega^2 [M]) \{\delta\} = 0.$$  

(19)

3.4. Continuity between adjacent laminas and delamination description

For an arbitrary laminated plate with an internal delamination as shown in Fig. 3, suppose that the delamination region is between two separate sublaminates called the upper and lower sublaminates, respectively. To ensure the material continuity, displacements and their variations of each pair of coincident nodes on any two upper and lower adjacent laminas have to be equal except those in the delaminated region.

To simulate the actual status of delamination, an extremely thin layer is inserted between the upper and lower sublaminates within the delaminated region. When the plate is in motion, it is physically impossible that elements of the upper and lower sublaminates penetrate into each other within the delamination region. Therefore, virtual elements are inserted between the penetrated nodes of the upper and lower sublaminates within the delaminated region to ensure a reasonable deformation without penetration.

Virtual/artificial spring elements have been previously used to simulate various conditions of structures [26] and the mechanical coupling between different components [27,28]. Further studies extended their use to vibroacoustic systems in which a structure is coupled to an acoustic medium [29]. This technique is used here to bring desirable effect to the interface in the delaminated region. Every inserted virtual element is a spring with a stiffness constant $k_r$. The local and global coordinate systems are $x_1, y_2, z_3$ and $x, y, z$, respectively as shown in Fig. 4. If the displacements of nodes $i$ and $j$ are $u_i, v_i, w_i$ and $u_j, v_j, w_j$, respectively, in the global coordinates the components of the distance change between the two nodes of a virtual element are expressed in terms of nodal displacements as

$$\{\delta_r\} = (u_j - u_i, v_j - v_i, w_j - w_i)^T = [B_r] \{\delta^e\}$$

(20)

where

$$[B_r] = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

The internal spring force can be expressed as

$$\{N_r\} = (N_{rz}, N_{ry}, N_{rz})^T = [A_r][K_r][A_r]^{-1} \{\delta_r\}$$

(21)

where

$$[K_r] = \begin{bmatrix} k_r & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(22)

is the local stiffness matrix of the spring with constant $k_r$, and

$$[A_r] = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Fig. 3. Geometry of multi-layer composite plate with local internal delamination.

Fig. 4. Virtual spring element and the coordinate systems.
is the transformation matrix between the global and local coordinate systems.

Let

$$[D_i] = [A_i][K_i][A_i]^{-1}$$

the nodal forces of the virtual element can be expressed as

$$\{F_i\} = (F_{ix}, F_{iy}, F_{iz})^T = [B_i]^T[D_i][B_i]\{\delta^i\}$$

Hence the stiffness matrix of the virtual element is

$$[K^v] = [B_i]^T[D_i][B_i]$$  \hspace{1cm} (23)

The spring constant $k_r$ in Eq. (22) will be so selected that the upper and lower parts within the delaminated region can keep a reasonable gap or contact without penetration when the plate is in motion. It will be discussed later.

### 4. Numerical examples and results

The proposed finite element formulation is then incorporated into a finite element analysis computer program for validation. Some examples are investigated numerically to analyze the variation of the natural frequencies and modal displacements and modal strains induced by delamination.

#### 4.1. Sample description

The samples for numerical simulation are multi-layer square plates composed of different fiber reinforced laminae with four edges free. Assume that all samples are orthotropic.

<table>
<thead>
<tr>
<th>Plate 1</th>
<th>Element number for each lamina</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>1</td>
<td>85.94</td>
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<tr>
<td>2</td>
<td>130.58</td>
</tr>
<tr>
<td>3</td>
<td>217.04</td>
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<td>4</td>
<td>417.07</td>
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<tr>
<td>5</td>
<td>729.89</td>
</tr>
<tr>
<td>6</td>
<td>765.70</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Plate 2</th>
<th>Element number for each lamina</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>97.56</td>
</tr>
<tr>
<td>2</td>
<td>174.70</td>
</tr>
<tr>
<td>3</td>
<td>278.83</td>
</tr>
<tr>
<td>4</td>
<td>305.11</td>
</tr>
<tr>
<td>5</td>
<td>328.38</td>
</tr>
<tr>
<td>6</td>
<td>521.88</td>
</tr>
</tbody>
</table>
Plates 1 and 2 for different meshes for one lamina. It is seen that the results converge after each lamina is discretized by 400 or more elements. Therefore, in order to save CPU time, 400 elements are adopted for each lamina in the subsequent computations except that the delamination region is fractionized.

Table 2 shows the natural frequencies of the first six modes for bending vibration of Plates 1 and 2 while the referential numerical and experimental results from Ref. [30] are also listed for comparison. It demonstrates a better agreement between the present results and the experimental results in Ref. [30].

### Table 2

<table>
<thead>
<tr>
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<td>82.26</td>
<td>83.57</td>
<td>95.66</td>
<td>108.17</td>
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<tr>
<td>2</td>
<td>113.10</td>
<td>118.42</td>
<td>152.53</td>
<td>168.64</td>
</tr>
<tr>
<td>3</td>
<td>207.29</td>
<td>207.79</td>
<td>238.74</td>
<td>218.64</td>
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<tr>
<td>4</td>
<td>325.28</td>
<td>329.41</td>
<td>258.03</td>
<td>280.15</td>
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<tr>
<td>5</td>
<td>408.51</td>
<td>419.83</td>
<td>277.50</td>
<td>301.00</td>
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<tr>
<td>6</td>
<td>539.92</td>
<td>546.93</td>
<td>496.54</td>
<td>505.15</td>
</tr>
</tbody>
</table>

#### 4.2.2. Effect of spring constant on natural frequency

The distance change of the adjacent points just above and below the delamination region may be different if the value of $k_r$ in Eq. (22) varies for the delaminated plate. In fact, the virtual spring with certain stiffness can introduce additional constraint, which may result in increase of vibration frequency [26–28]. Without such inserted virtual spring elements, the movements of nodes on the free surface of the delamination will not be restricted. With nearly complete restriction, i.e. the spring constant of the virtual element being extremely large (e.g. larger than $10^6$ N m$^{-1}$), the natural frequencies of the plate will increase significantly especially for high modes with large delamination.

Fig. 5 shows the displacements of the upper and lower points within the delaminated region for the first 10 modes of Plate 3 with delamination areas of 165 and 2475.5 mm$^2$, respectively, when the virtual spring element is not inserted. The names and locations of the above points are listed in Table 3. It is apparent that penetration between the upper and lower parts within the delaminated region occurs in modes 6 and 7 when delamination area is 165 mm$^2$, and for the case of delamination area 2475.5 mm$^2$ penetration is obviously seen in the sixth, seventh and ninth modes.

Further numerical simulation for Plate 3 with the delamination areas of 165 and 2475.5 mm$^2$ shows that when $k_r$ is larger than 0.1 N m$^{-1}$ the upper and lower sublaminates in the delamination region do not penetrate into each other for the above three modes when the plate is in motion. Variations of natural frequencies with different values of $k_r$ are shown in Table 4 for the first 10 modes of Plate 3 with the delamination area of 2475.5 mm$^2$. It is seen that the spring-induced increase of frequency is not significant when the spring constant is less than $10^3$ N m$^{-1}$. Fig. 6 shows the decreases of natural frequencies of the first 15 modes for Plate 3 with different delamination areas without the virtual spring elements in the delaminated region. Table 5 shows the influence of spring constant on natural frequencies of the sixth, seventh and ninth modes for Plate 3 with different delamination areas when $k_r = 10^6$ N m$^{-1}$.

A combination of comparison between Fig. 5 and Table 5 with Table 4 reveals that the effect of the spring
constant on natural frequency increases with the delamination area and becomes relatively remarkable when the value of $k_r$ goes up. Therefore, virtual spring elements with spring constant $k_r = 10^{-1}$ Nm$^{-1}$ are adopted for numerical computation of modes 6, 7 and 9 of the delaminated plates. The variations of natural frequencies with the delamination areas are listed in Table 6 for the above three modes when $k_r$ is $10^{-1}$ Nm$^{-1}$. It is seen that although the effect of virtual spring element increases when the delamination grows, but it is not as significant as that of delamination.

4.2.3. Effect of delamination area on natural frequencies

The effect of delamination area on natural frequencies of the multi-layer composite plate is studied numerically for Plate 3. As shown in Fig. 3, the delamination region is assumed elliptic with center C. From Fig. 6 and Table 6, it can be seen that the natural frequency decreases with the increase of delamination area. The only exception occurs when the delamination area is 165 mm$^2$ (the smallest area for analysis) for mode 6. This may be caused by the introduction of the virtual spring elements because the location of the delamination is just near the peak of mode shape for mode 6 as shown in Fig. 7. But even in this case the percentage change of natural frequency is less than 0.05%. Therefore errors introduced by the virtual element will not become a menace in study on delamination-induced variations of other parameters more applicable than frequency, such as displacement and strain.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Spring constant $k_r$ (Nm$^{-1}$)</th>
<th>$10^{-2}$</th>
<th>$10^{-1}$</th>
<th>$10^2$</th>
<th>$10^3$</th>
<th>$10^4$</th>
<th>$10^5$</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>0</td>
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<td>0</td>
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<td>7</td>
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<td>0</td>
<td>1.04</td>
<td>6.1</td>
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From Fig. 6 and Table 6, it can be seen that the natural frequency generally decreases with the increase of delamination area. The only exception occurs when the delamination area is 165 mm² (the smallest area to be analyzed) for mode 6. This may be caused by the introduction of the virtual spring elements because the location of the delamination is very near the peak of mode shape for mode 6 as shown in Fig. 7. However, even in this case the percentage change of natural frequency is less than 0.05%. Therefore, errors introduced by the virtual elements cannot become a menace in study on delamination-induced variations of other parameters more applicable than frequency, such as displacement and strain.

Table 6 shows that delamination with small area has very little influence on natural frequencies for all computed modes. The average percentage change of natural frequencies is only about 0.066% when the delamination area reaches 990.2 mm², and even the maximum value is about 0.18% for mode 10. A similar variation pattern is also seen when the delamination area increases gradually. The frequency changes are greater than 1% for modes 11 and 12, and the maximum is about 1.16% for the fifth mode when the delamination area is 2475.5 mm². This implies that the natural frequency is not sensitive to delamination for multi-layer composite plates.

The above results show that the influence of delamination on natural frequency varies with vibration modes, and this phenomenon may be useful to determine the location or area of delamination. However, this is basically impracticable, because, the experimental equipment can hardly extract modes higher than 10 for composite plates, and it is difficult to measure the small delamination-induced change of frequency.

If internal delamination occurs in the vibrating plate, there may be interactive motion or impact between the upper and lower sublaminates within the delamination region. Further investigation on the local displacements of the plate shows that the differences of displacements between the points just on the upper and lower surfaces of the delamination region vary with modes. The relative displacements in x-y plane are also larger in some modes than in other modes, i.e., the interactive motion between the upper and lower surfaces within the delamination region occurs when the plate vibrates. Thus, when internal delamination occurs somewhere in a composite plate, there may be interactive motion or impact within the delamination region during vibration of the plate. These phenomena cause the variations of energy dissipation in the plate. Hence, the variation of energy dissipation in the plate during vibration can be the hint for delamination detection. Therefore, the relative displacement in z direction within the delamination region is restricted using virtual spring to avoid the physically impossible penetration, and the selection of \( k_c \) with respect to the transverse properties of the orthotropic lamina is not necessary.
4.2.4. Variation of displacement and strain due to delamination of the vibrating composite plate

Analysis of modal parameters is known as one of the practical experimental methods for damage detection. As delamination of multi-layer composites means local structural change, it can, to some extent, induce variations of vibration responses. To investigate the delamination-dependent vibration responses the changes of modal deformation have been computed for delaminated composite of Plate 3. For the first 15 modes the modal displacements along $z$-direction and modal strains along $x$-direction and $y$-direction are computed respectively at some concerned points.

For the first 15 modes, displacements and strains are computed at the point with the same $x$ and $y$ coordinates as those of point C as shown in Fig. 3, and $z = 2.05$ mm, i.e. the point on the top free surface of Plate 3. Fig. 8 shows the results of relative changes of frequencies, peak displacements and strains when the delamination area is 165.0 mm$^2$ for Plate 3. In the table $w_m$ represents the unit-normalized displacement along $z$ axis at the considered point.

Fig. 8 shows that delamination-induced changes of displacement and strain are much greater than that of frequency. The changes of displacement and strain are more remarkable in the third, fifth, sixth and seventh modes than those in other modes. Both the most remarkable changes of displacement and strain appear in the same mode—the third mode. It should be noted that the most remarkable decrease of frequency also occurs in the third mode for the computed case. From the third mode shape of Plate 3 as shown in Fig. 9, it is noted that the delamination is close to the region of plate displacements transition from positive to negative.

![Fig. 8. Relative changes of modal parameters at a point on top surface just above the center of the delamination region for Plate 3 with a delamination area of 165.0 mm$^2$.](image)

![Fig. 9. The third displacement mode shape for Plate 3.](image)

**Table 7**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Coordinate of point for computation (mm)</th>
<th>Relative change of $w_m$ (%)</th>
<th>Relative change of $\varepsilon_{xx}$ (%)</th>
<th>Relative change of $\varepsilon_{yy}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 0 2.05</td>
<td>-0.02</td>
<td>-0.051</td>
<td>-0.049</td>
</tr>
<tr>
<td>2</td>
<td>112.75 225.5 2.05</td>
<td>0</td>
<td>0.119</td>
<td>0.123</td>
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<tr>
<td>3</td>
<td>225.5 225.5 2.05</td>
<td>0.001</td>
<td>-13.488</td>
<td>-1.085</td>
</tr>
<tr>
<td>4</td>
<td>225.5 0 2.05</td>
<td>0</td>
<td>0.388</td>
<td>0.204</td>
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<tr>
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<td>0 225.5 2.05</td>
<td>-1.929</td>
<td>-1.543</td>
<td>-1.061</td>
</tr>
<tr>
<td>6</td>
<td>225.5 0 2.05</td>
<td>-14.081</td>
<td>-19.229</td>
<td>-7.727</td>
</tr>
<tr>
<td>7</td>
<td>0 66.625 2.05</td>
<td>-13.154</td>
<td>-15.299</td>
<td>-5.614</td>
</tr>
<tr>
<td>8</td>
<td>0 10.25 2.05</td>
<td>-0.665</td>
<td>0.183</td>
<td>-0.219</td>
</tr>
<tr>
<td>9</td>
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<td>-0.001</td>
<td>-0.384</td>
<td>-0.291</td>
</tr>
<tr>
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<td>0.134</td>
<td>0.127</td>
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<tr>
<td>11</td>
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<td>0</td>
<td>-0.109</td>
<td>-0.111</td>
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<tr>
<td>12</td>
<td>30.75 0 2.05</td>
<td>-0.27</td>
<td>-0.062</td>
<td>-0.064</td>
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<tr>
<td>13</td>
<td>225.5 225.5 2.05</td>
<td>0</td>
<td>0.036</td>
<td>0.522</td>
</tr>
<tr>
<td>14</td>
<td>225.5 153.75 2.05</td>
<td>-0.714</td>
<td>0.04</td>
<td>0.217</td>
</tr>
<tr>
<td>15</td>
<td>112.75 0 2.05</td>
<td>0.001</td>
<td>0</td>
<td>-0.217</td>
</tr>
<tr>
<td>Average of absolute values</td>
<td>2.059</td>
<td>3.404</td>
<td>3.131</td>
<td></td>
</tr>
</tbody>
</table>

In order to investigate the local variation of deformation induced by delamination, changes of displacements and strains at another point are computed for the considered modes. Table 7 shows the relative changes of displacements and strains at the point with the maximum displacement on the top surface of Plate 3 before delamination. It can be seen that unlike the results in Fig. 8, neither the displacement change nor the strain change is the largest for the third mode although this point (with coordinates 225.5, 225.5, 2.05) in Table 7 is not far away from the point, the results of which are shown in Fig. 8.

Therefore, the influence of delamination on deformation of a composite plate is dependent on modes. It is an effective method to detect damage by investigating into damage-induced local changes of structural geometric, physical and mechanical parameters by means of displacement and strain measurement with consideration of natural frequency change. In fact, experimental modal strain analysis has already been proved as reasonable and practical for investigation of local variation of structural parameters [31].

5. Conclusions

The delamination problem for multi-layer composite plates has been analyzed using finite element method and modal analysis. To achieve accurate results for delamination detection of multi-layer composite plate, different fiber orientations, orthotropy of laminated composites and transverse shear effect are taken into account in the finite element computation. For practical nondestructive damage detection, internal elliptic delamination is considered and virtual interface elements are introduced to simulate local delamination and to prevent physically impossible penetration. The following conclusions may be drawn from the results of numerical simulation in this study.

(a) The finite element model proposed in this paper can predict accurately the dynamic behaviors of a multi-layer composite plate with internal delamination at arbitrary location. It is not as computationally expensive as the usually used three-dimensional brick elements when the proposed element is used because there is no restriction of ratio between element length and thickness. The virtual element with very small spring constant has very slight influence on natural frequencies and mode shapes of the computed plates but can successfully prevent physically impossible penetration of the upper and lower parts within the delaminated region.

(b) Local internal delamination has slight effect on natural frequencies of a multi-layer composite plate although the extent of natural frequency variation increases with both the delamination dimension and the order of natural frequency.

(c) Delamination-induced change of deformation is more sensitive than that of frequency, and changes of both displacement and strain are mode-dependent. The most remarkable delamination-induced changes of displacement and strain occur in the same mode as the most remarkable decrease of frequency when the delamination area is relatively small for the cases considered in this paper.

(d) The results of numerical analysis in this study can be taken as guidance for arrangement of displacement and strain measurements on the surface of specimen when experimental modal analysis is carried out to investigate local changes of structural parameters for damage detection. The finite element model, strategy and numerical method provided in this paper can be used for dynamic response analysis of damaged engineering structures, especially for multi-layer composite plates.

(e) If the layers are also damaged, for example cracks occur in a laminar, the damage in the related elements must be considered via describing the variation of material elastic parameters. In this case, the elastic matrix of damaged element is not the same as that of the intact one. By introducing special variables into the elastic matrix to indicate the damage features, the dynamic behaviors of the damaged composites can be computed, and the damage-induced variations of modal parameters can then be investigated for damage detection. This topic is also under research in our project.

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References