Reynolds Number Effects on Three-Dimensional Vorticity in a Turbulent Wake

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When Reynolds number Re (≡ U∞d/ν, where U∞ is the free stream velocity, d is the cylinder diameter, and ν is the kinematic viscosity of fluid) varies from 10^3 to 10^4, there is a large change in the turbulent near-wake dynamics (e.g., the base pressure coefficient, fluctuating lift coefficient, and vortex formation length) of a circular cylinder, which has previously been connected to the generation of small-scale Kelvin–Helmholtz vortices. This work aims to investigate how this Re variation affects the three components of the vorticity vector and to provide a relatively complete set of three-dimensional vorticity data. All three components of vorticity were simultaneously measured in the intermediate region of a turbulent circular-cylinder wake using a multiwire vorticity probe. It is observed that the root-mean-square values of the three vorticity components increase with Re, especially the streamwise component, which shows a large jump from Re = 5 × 10^3 to Re = 10^4. At Re = 2.5 × 10^4, the maximum phase-averaged spanwise vorticity variance (ω_z^2)^½, normalized by d and U∞, is twice as large as its counterpart for the streamwise component, (ω_x^2)^½, or the lateral component, (ω_y^2)^½. However, at Re = 10^4, the maximum (ω_z^2)^½ is only 55% larger than the maximum (ω_x^2)^½ or 47% larger than the maximum (ω_y^2)^½. The observation is consistent with the perception that the three-dimensionality of the flow is enhanced at higher Re due to the occurrence of Kelvin–Helmholtz vortices. The effect of Re on vorticity signals, spectra, and coherent and incoherent vorticity fields is also examined.

I. Introduction

Flow around a circular cylinder has been a subject of intensive interest to engineers and scientists for many decades. The Reynolds number, Re (≡ U∞d/ν, where U∞ is the free stream velocity, d is the cylinder diameter, and ν is the kinematic viscosity of fluid), has a profound effect on the near-wake dynamics. This effect has attracted a great deal of attention in the literature, particularly in the range of Re = 10^3–10^4. In this range, the Strouhal number Sr and the drag coefficient C_d experience mild variation. However, there is a large change in the formation length of Kármán vortices, and the mean base pressure coefficient −C_p exhibits an almost 50% increase. The fluctuating lift coefficient C_l displays a drastic increase from nearly zero at Re = 1.0 × 10^3 to about 0.4 at Re = 1.0 × 10^4 (Refs. 1 and 4–6). Accordingly, the velocity fluctuation in the shear layer around the cylinder increases significantly. Instantaneous structures show the generation of small-scale Kelvin–Helmholtz vortices, particularly evident at Re = 5 × 10^3 and even more so at Re = 1.0 × 10^4 (Ref. 8). One interesting question to be asked is how these near-wake variations with Re would impact upon the downstream wake. This work aims to investigate the Re effect on the evolution of the three-dimensional vorticity, which is an important characteristic of turbulence, in the intermediate wake.

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All of these terms can be measured by the present probe. The values of $\varepsilon$ obtained from Eq. (1c) are used to calculate the Kolmogorov length scale $\eta$. These values are given in Table 1. The separation between the two inclined wires of each X wire was about 0.6 mm. Another X wire, placed at $y = 4 – 7d$, depending on the measurement station $x/d = 10 – 40$, was used in conjunction with the vorticity probe in order to provide a phase reference for the measured vorticity signals (Fig. 1c).

The hot wires were etched from Wollaston (Pt–10% Rh) wires. The active length was about 200$d_w$, where $d_w = 2.5 \mu m$ is the wire diameter. The wires were operated on in-house constant-temperature circuits at an overheating ratio of 0.5. The probe was calibrated at the centerline of the tunnel using a pitot static tube connected to a MKS Baratron pressure transducer (least count $= 0.01$ mm H$_2$O). The yaw calibration was performed over ±20 deg. The included angle of each X wire was about 10 deg and the effective angle of the inclined wires was about 35 deg, which was sufficient to minimize the effect of large velocity cone angles (e.g., Perry et al.19 and Browne et al.20). Output signals from the anemometers were passed through buck and gain circuits and low-pass filtered at a cutoff frequency $f$, close to $U/2\pi$, which is commonly identified as the Kolmogorov frequency $f_K$, where $U$ is the local mean velocity in the streamwise direction. The filtered signals were subsequently sampled at a frequency $f_s = 2f$, using a 12-bit analog-to-digital converter. The duration of data was about 60 s.

It is assumed that each X probe measures the two velocity components at the center of the probe. While measured velocity components can be significantly in error when the velocity gradients are steep,16,21 the mean velocity gradient is not significant in the present flow. Using a range of hot-wire yaw factors corresponding to these experimental conditions, errors in neglecting the fluctuating instantaneous velocity gradients are estimated to be about 3% and 4% for $u_{rms}$ and $v_{rms}$ (or $w_{rms}$), respectively, where $u$, $v$, and $w$ are the velocity fluctuations in the $x$, $y$, and $z$ directions, respectively. The binormal cooling effect on each X probe has also been neglected, which only gives rise to an error of 1–3% for $u_{rms}$ and $v_{rms}$ (or $w_{rms}$) when the local turbulence intensity is about 10%. The present $u_{rms}/U$ ranges from 20% to 10% for $x/d = 10–40$. Therefore, the error in neglecting the binormal cooling effect is estimated to be about 4%. Experimental uncertainties in $U$ and $u_{rms}$ (or $v_{rms}$ and $w_{rms}$) were inferred from errors in the hot-wire calibration data as well as the scatter (20 to 1 odds) observed in repeating the experiment a number of times. The uncertainty for $U$ was about ±2%, while uncertainties for $u_{rms}$, $v_{rms}$, and $w_{rms}$ were about ±5%, ±6%, and ±6%, respectively.

The vorticity components are calculated from the measured velocity signals, viz.,

$$ \omega_x = \frac{\partial w}{\partial z} - \frac{\partial u}{\partial y} \approx \frac{\Delta w}{\Delta z} - \frac{\Delta u}{\Delta y} \quad (2) $$

$$ \omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \approx \frac{\Delta u}{\Delta z} - \frac{\Delta w}{\Delta x} \quad (3) $$

$$ \omega_z = \frac{\partial v}{\partial x} - \frac{\partial (U + w)}{\partial y} \approx \frac{\Delta v}{\Delta x} - \frac{\Delta (U + w)}{\Delta y} \quad (4) $$

where $\Delta u$ and $\Delta u$ in Eqs. (2) and (4) are velocity differences between X wires $a$ and $c$ (Figs. 1a and 1b); $\Delta v$ and $\Delta u$ in Eqs. (2) and (3) are velocity differences between X wires $b$ and $d$. The spatial separation $\Delta x$ is estimated based on Taylor’s hypothesis, given by $-U/(2\Delta t)$, where $U$ is the average convection velocity of vortices, given by $0.87 \sim 0.89U_\infty$, depending on the streamwise location.24

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**Table 1: Kolmogorov length scales (millimeters) at various locations and Reynolds numbers**

<table>
<thead>
<tr>
<th>$x/d$</th>
<th>$Re = 2.5 \times 10^3$</th>
<th>$Re = 5.0 \times 10^3$</th>
<th>$Re = 1.0 \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.17</td>
<td>0.12</td>
<td>0.076</td>
</tr>
<tr>
<td>20</td>
<td>0.22</td>
<td>0.14</td>
<td>0.09</td>
</tr>
<tr>
<td>40</td>
<td>0.31</td>
<td>0.2</td>
<td>0.12</td>
</tr>
</tbody>
</table>
and $\Delta t = 1/f$, is the time interval between two consecutive points in the time series of velocity signals. This method would have the advantages of allowing $\Delta x \approx \Delta y \approx \Delta z$ to avoid phase shift and to maintain the same level of spectral attenuation for the same Reynolds number between the velocity gradients occurring in Eqs. (2–4). Because of limited spatial resolution of the probe, the velocity derivatives in the vorticity components are expected to be underestimated. The errors in measuring vorticity components at $x/d = 10, 20, \text{and} 40$ are estimated to be about $28\%$, $18\%$, and $10\%$ for $Re = 2.5 \times 10^3$, $41\%$, $33\%$, and $22\%$ for $Re = 5.0 \times 10^3$, and $68\%$, $55\%$, and $41\%$ for $Re = 1.0 \times 10^4$, respectively.

III. Characteristics of the Three Vorticity Components

A. Vorticity Signals

Vorticity is derived from velocity fluctuations measured by the four X wires. Measured velocity signals may be erroneous if the probe is not constructed properly, especially because of inappropriate effective angles. To examine the appropriateness of effective angles, the probability density functions (pdfs) of the velocity vector angle ($\beta$) at $x/d = 10$ for different Reynolds numbers are calculated and shown in Fig. 2. The velocity vector angle is calculated based on the instantaneous longitudinal and transverse velocity components, $U$ and $V$. For all Reynolds numbers, the velocity vector angles are comparable. The probability that the velocity vector angles exceed $\pm 35$ deg is only $0.1\%$, resulting in a negligible effect on measured velocity fluctuations. Note that $\beta$ may be larger than the calibration yaw angle ($\pm 20$ deg). Ideally, the yaw angle should exceed the largest $\beta$. However, Browne et al. found that the dependence of the effective angle of the X wire on the yaw angle was negligible over the range from $-40$ to $15$ deg and concluded that the calibration yaw angle range of $\pm 15$ deg is adequate for flows with a moderate turbulence intensity. Furthermore, for all $Re$ investigated, only $8\%$ of $\beta$ for $y/d \leq 1.0$ are larger than the maximum yaw angle ($\pm 20$ deg). The percentage drops to $2\%$ for a larger $x/d$. It is therefore concluded that the errors due to a relatively small range of yaw angles for calibration may be negligible.

Figure 3 presents the instantaneous signals of $\omega_x$, $\omega_y$, and $\omega_z$, along with that of $v$, near the most likely vortex location ($y/d = 0.6$) at $x/d = 10$ for $Re = 2.5 \times 10^3$, $5.0 \times 10^3$, and $1.0 \times 10^4$, where an asterisk denotes normalization by $d$ and $U_\infty$. (The arrow indicates the direction of the flow. The dots and dashed vertical lines on the $v$ signals indicate possible vortex centers. The horizontal lines are zero levels of the signals.) The $v$- and $\omega_z$-signals for all $Re$ display large-scale quasi-periodical fluctuations, indicating the occurrence of Kármán vortices. The zero $v$ of positive $dv/dt$ generally corresponds to the large-scale $\omega_z$-fluctuations of negative sign and is identified with the possible Kármán vortex center, marked by a dot in the $v$-signal. The fluctuations, near the vortex centers, of the $\omega_x$- and $\omega_y$-signals conform to the three-dimensionality of Kármán vortices. The $\omega_y$- and $\omega_z$-signals display significant fluctuations in either sign between spanwise vortex centers, which could be linked to the occurrence of quasi-longitudinal rib structures. As $Re$ increases, small-scale fluctuations in the vorticity signals increase, particularly evident as $Re$ changes from $5.0 \times 10^3$ to $1.0 \times 10^4$. The observation is consistent with the previous report, based on high-image-density particle-image velocimetry, that small-scale Kelvin–Helmholtz vortices were evident at $Re = 5.0 \times 10^3$ and more so at $Re = 1.0 \times 10^4$. It could be inferred that the Kelvin–Helmholtz vortices are partially responsible for the increased three-dimensionality of the flow.

B. Probability Density Function

Figure 4 shows the pdfs, $P(\omega_x)$, $P(\omega_y)$, and $P(\omega_z)$, calculated based on vorticity signals measured near the most probable vortex path ($y/d = 0.6$) at $x/d = 10$. The abscissa has been normalized by the root-mean-square vorticity. $P(\omega_x)$ (Fig. 4a) is quite symmetric about zero for $Re = 2.5 \times 10^3$ and $5.0 \times 10^3$ but shows a slightly skewed peak for $Re = 1.0 \times 10^4$. This skewness is probably
due to experimental errors, because $\omega_x/\omega_{x, rms}$ must be symmetric about zero as a result of the two-dimensional mean flow. $P(\omega_x)$ (Fig. 4b) displays its peak at $\omega_x/\omega_{x, rms} = 0$. The peak in $P(\omega_y)$ and $P(\omega_z)$ at $Re = 1.0 \times 10^6$ drops considerably. This is more evident for $P(\omega_x)$ and $P(\omega_y)$ at $Re = 1.0 \times 10^6$, broadening appreciably for $\omega_x/\omega_{x, rms} < \pm 2$ if compared with $Re = 2.5 \times 10^5$ and $5.0 \times 10^5$. The observation is internally consistent with the increasing relatively small-scale fluctuations (Fig. 3) as $Re$ increases. Presumably, the broadening is largely linked to the increased Kelvin–Helmholtz vortices. The more appreciable broadening in $P(\omega_x)$ than in $P(\omega_y)$ suggests that Kelvin–Helmholtz vortices tend to be aligned stream-wise, consistent with direct numerical simulation. This further suggests a connection between Kelvin–Helmholtz vortices and the rib structures, which are generally considered to be quasi-longitudinal\(^*\) (Fig. 5). $P(\omega_x)$ exhibits twin peaks (Fig. 4c). The peak of negative vorticity is due to the spanwise vortices of negative sign that occur at $y/d < 0$, whereas that of positive vorticity is attributed to the spanwise vortices of positive sign, whose center is at $y/d > 0$. At $x/d = 10$, the centers of both positive and negative vortices occur near the centerline (e.g., Zhou et al.\(^*\)). Therefore, the vortices of both signs can be detected at $y/d = 0.6$. As $Re$ increases from $2.5 \times 10^5$ to $1.0 \times 10^5$, $\omega_x/\omega_{x, rms}$ corresponding to the negative peak approaches zero, moving from $-0.47$ to $-0.20$ (mostly between $Re = 5.0 \times 10^5$ and $1.0 \times 10^5$). This variation suggests a reduction in the most likely spanwise vorticity magnitude at the vortex center. Accordingly, the time-averaged normalized spanwise vorticity (not shown) decreases in magnitude as $Re$ increases. The three-dimensionality, along with the viscous dissipation, mixing of opposite-signed vorticities, entrainment, and nonlinear effects, may cause a loss of spanwise vorticity.\(^{21}\) The three-dimensionality results from the slant of spanwise vortex rolls and the occurrence of predominantly longitudinal structures such as Kelvin–Helmholtz vortices. The assertion is corroborated by faster growing rms values, $\omega_{x, rms}$ and $\omega_{y, rms}$, relative to $\omega_{z, rms}$ as discussed in the next subsection.

C. Reynolds Stresses and rms Vorticity

The normalized Reynolds stresses across the wake, $u'^2$, $v'^2$, and $w'^2$, are shown in Fig. 6 for $x/d = 10-40$. The Reynolds stresses in a circular cylinder wake have been extensively documented in the literature\(^{12,13,28,30,32-35}\). The near wake of a circular cylinder is developing and the Reynolds normal stresses increase with $Re$. As a result, the agreement in $u'^2$ and $v'^2$ between the present and previous data is good for comparable $Re$ but not so for different $Re$. The $Re$ dependence of the Reynolds normal stresses appear less evident for a larger $x/d$. On the other hand, the Reynolds shear stress exhibits little dependence on $Re$.

Figure 7 presents the cross-flow distribution of the rms values, $\omega_{x, rms}$, $\omega_{y, rms}$, and $\omega_{z, rms}$, of the three vorticity components. The ratio of rms vorticity values at the centerline and $y/d \approx 7$, which may be considered to be approximately in the free stream, is between 40 and 150 for different $Re$, indicating a good signal-to-noise ratio. For the purpose of comparison, previous measurements\(^{9,12,13}\) are also included in the figure. Zhang et al.\(^{15}\) measurement was conducted at $Re = 5.6 \times 10^5$. But, due to poor spatial resolution (3.5 $\sim$ 5 mm), their data are even smaller than the present measurement at $Re = 2.5 \times 10^5$. Mi and Antonia\(^{12}\) used a four-wire probe (consisting of two X wires separated by a distance of 1.2 mm) and applied a correction to the measured rms values. As a result, their $\omega_{x, rms}$ and $\omega_{y, rms}$ were considerably larger than the present data at $Re = 2.5 \times 10^5$. Marasli et al.\(^{9}\) procured their measurement at $x/d = 30$ using a 12-wire probe with a spatial resolution of about 2 mm. Because vorticity experiences fast streamwise decay, the difference in $x/d$ along with a lower $Re (=2.0 \times 10^5)$ contributes to the
observation that their results are smaller than the present ones. This comparison indicates that the present vorticity measurement is reasonable. The values of \( \omega_{rms}^x \), \( \omega_{rms}^y \), and \( \omega_{rms}^z \) rise with increasing \( Re \). In contrast with \( u^2 \), \( v^2 \), and \( w^2 \), the \( Re \) dependence of \( \omega_{rms}^x \), \( \omega_{rms}^y \), and \( \omega_{rms}^z \) persists even at \( x/d = 40 \). At \( x/d = 10 \) (Figs. 7a, 7d, and 7g), \( \omega_{rms}^x \) and \( \omega_{rms}^z \) are considerably smaller than \( \omega_{rms}^y \) for \( Re \leq 5.0 \times 10^5 \). However, \( \omega_{rms}^x \) and \( \omega_{rms}^z \) show a faster growth rate than \( \omega_{rms}^y \), \( \omega_{rms}^x \), and \( \omega_{rms}^z \) increase by 29%, 27%, and 10%, respectively, from \( Re = 2.5 \times 10^4 \) to 1.0 \( \times 10^5 \). As a matter of fact, the \( \omega_{rms} \) values of the three vorticity components are approximately the same at \( Re = 1.0 \times 10^5 \), which is agreeable with Zhang et al.’s report. As illustrated in Fig. 5, small-scale Kelvin–Helmholtz vortices or rib structures are predominantly aligned in the streamwise direction. The fact that most of the increase in \( \omega_{rms} \) and \( \omega_{rms} \) occurs from \( Re = 5.0 \times 10^5 \) to 1.0 \( \times 10^5 \) is consistent with the intensified generation of Kelvin–Helmholtz vortices in this \( Re \) range.

IV. Coherent Vorticity Field

A. Phase Averaging

The coherent vorticity field may be extracted using a phase-averaging method. Briefly, the \( v \)-signals from the moving vorticity probe and fixed X wire were both digitally bandpass filtered with the center frequency set at \( f_x \), estimated from the frequency at which a pronounced peak occurred in the \( v \)-spectrum, using a fourth-order Butterworth filter. The low- and high-pass frequencies were chosen to be the same as \( f_x \), that is, a zero bandpass width was chosen. Such a choice allows a better focus on the organized structures.

The filtered signal \( v_f \) is given by the thicker line in Fig. 8. Two phases of particular interest are identified on \( v_f \), viz.

\[
\begin{align*}
\text{Phase A: } & \frac{dv_f}{dt} > 0, \quad v_f = 0 \quad (5) \\
\text{Phase B: } & \frac{dv_f}{dt} < 0, \quad v_f = 0 \quad (6)
\end{align*}
\]

B. Extract of Coherent Structure

Based on the triple decomposition and the variable \( B \) can be viewed as the sum of the time mean \( \bar{B} \) and the fluctuating component \( \beta \):

\[
B = \bar{B} + \beta \quad (10)
\]

where \( B \) stands for instantaneous vorticity. The fluctuation \( \beta \) may represent \( \omega_x, \omega_y, \) or \( \omega_z \) and can be further decomposed into the coherent fluctuation \( \beta \equiv \langle \beta \rangle \) and a remainder \( \beta', \) viz.

\[
\beta = \bar{\beta} + \beta' \quad (11)
\]

The coherent fluctuation \( \bar{\beta} \equiv \langle \beta \rangle \) reflects the effect from the large-scale coherent structures, while the remainder \( \beta' \) mainly results from the incoherent structures. Squaring to both sides of Eq. (11) and assuming a negligible correlation between the coherent fluctuation and the remainder yields the equation

\[
\langle \beta^2 \rangle = \bar{\beta}^2 + \langle \beta'^2 \rangle \quad (12)
\]
C. Coherent Vorticity Fields

Figure 9 presents the iso contours of the phase-averaged or coherent vorticity, $\omega_0^c$, at $Re = 2.5 \times 10^4$. The phase $\phi$, ranging from $-2\pi$ to $+2\pi$, can be interpreted in terms of a longitudinal distance, $x/d = 2\pi$ corresponding to the average vortex wavelength. To avoid any distortion of the physical space the same scales are used in the $\phi$ and $y'$ directions in Figs. 9 and 10. The positions of centers and saddle points, estimated from sectional streamlines (not shown), are denoted by + and \( \times \), respectively. The $\omega_0^c$ contours (Fig. 9) display the familiar Kármán vortex street (the flow field below $y' < 0$ is not shown). The vortex center, identified by the maximum concentration of $\omega_0^c$ and marked by $+$, slowly shifts toward the free stream for increasing $x/d$, from $y'/d \approx 0.5$ at $x/d = 10$ to $y'/d \approx 1.0$ at $x/d = 40$. The result agrees with previous reports.\(^{18,21}\) The maximum concentration of $\omega_0^c$ impairs for a larger $x/d$. Table 2 lists the maximum magnitude of $\omega_0^c$ at $x/d = 10$, along with previous reports. The present data are in good agreement with Matsumura and Antonia’s estimate\(^{26}\) also obtained using a phase-averaging technique, but are smaller than Hussain and Hayakawa’s measurement.\(^{26}\) The difference may be ascribed to two reasons, different measurement techniques and detection techniques. The present vorticity data should have improved spatial resolution (about 2 mm), compared with Hussain and Hayakawa’s data (about 5 mm). Nonetheless, their detections based on vorticity concentrations probably contribute to the higher coherent vorticity concentration. It seems that the maximum $\omega_0^c$ is approximately independent of $Re$.

The present phase-averaging technique based on spanwise vortices is unable to separate coherent and incoherent $\omega_0$ or $\omega_1$, whose signs may be randomly given with respect to the spanwise vortex, resulting in the cancellation of positive and negative vorticity during phase-averaging. Therefore, the contours of phase-averaged vorticity variances $(\omega_0^c)^2, (\omega_0^{c*})^2,$ and $(\omega_1^{c*})^2$ at $x/d = 10$ are calculated. With a sufficiently large number of detections, the averaged $(\omega_0^c)^2, (\omega_0^{c*})^2,$ and $(\omega_1^{c*})^2$ over one wavelength of spanwise vortices should be equal to the corresponding time-averaged vorticity variances, i.e., $\omega_0^2, \omega_1^2,$ and $\omega_1^2$, respectively, which has been verified (the difference is within 10%). Therefore, the phase-averaged vorticity variances are effectively the sum of the coherent and incoherent structures.

### Table 2: Maximum magnitude of $\omega_0^c$ at different Reynolds numbers

<table>
<thead>
<tr>
<th>$Re$</th>
<th>$\omega_0^c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.5 \times 10^4$</td>
<td>1.09</td>
</tr>
<tr>
<td>$5.0 \times 10^3$</td>
<td>1.02</td>
</tr>
<tr>
<td>$5.83 \times 10^3$ (Ref. 33)</td>
<td>1.03</td>
</tr>
<tr>
<td>$1.0 \times 10^4$</td>
<td>0.94</td>
</tr>
<tr>
<td>$1.3 \times 10^4$ (Ref. 26)</td>
<td>1.09</td>
</tr>
</tbody>
</table>

*Fig. 9 Phase-averaged vorticity contours $\omega_0^c$: a) $Re = 2.5 \times 10^4$; $x/d = 10$, contour interval = 0.2; b) $Re = 2.5 \times 10^3$; 20, 0.1; c) $Re = 2.5 \times 10^3$; 40, 0.04; d) $Re = 5.0 \times 10^2$; 10, 0.2; e) $Re = 5.0 \times 10^2$; 20, 0.1; f) $Re = 5.0 \times 10^2$; 40, 0.04; g) $Re = 1.0 \times 10^4$; 10, 0.2; h) $Re = 1.0 \times 10^4$; 20, 0.1; and i) $Re = 1.0 \times 10^4$; 40, 0.04. (Centers and saddle points are denoted by $+$ and $\times$.)

*Fig. 10 Contours of phase-averaged vorticity variances at $x/d = 10$: a) $(\omega_0^c)^2$: $Re = 2.5 \times 10^4$; contour interval = 0.2; b) $(\omega_0^c)^2$: $Re = 2.5 \times 10^3$; 0.19; c) $(\omega_0^c)^2$: $Re = 5.0 \times 10^3$; 0.28; d) $(\omega_0^c)^2$: $Re = 5.0 \times 10^2$; 0.2; e) $(\omega_0^c)^2$: $Re = 5.0 \times 10^3$; 0.19; f) $(\omega_0^c)^2$: $Re = 5.0 \times 10^2$; 0.28; g) $(\omega_0^c)^2$: $Re = 1.0 \times 10^4$; 0.2; h) $(\omega_0^c)^2$: $Re = 1.0 \times 10^4$; 0.19; and i) $(\omega_0^c)^2$: $Re = 1.0 \times 10^4$; 0.28. (Centers and saddle points are denoted by $+$ and $\times$.)
The \( \omega_z^* \) contours (Fig. 10) display a pattern similar to that of \( \omega_x^* \), i.e., the Kármán vortex street, suggesting a predominant contribution from the spanwise structures. The thicker solid line denotes the outermost vorticity contours of Kelvin–Helmholtz vortices or quasi-longitudinal rib structures. At \( Re = 2.5 \times 10^4 \), the maximum \( \omega_z^* \) is twice as large as those of \( \omega_x^* \) and \( \omega_y^* \). However, as \( Re \) increases, the difference between the maximum \( \omega_x^* \) and \( \omega_z^* \) or \( \omega_y^* \) diminishes. At \( Re = 1.0 \times 10^5 \), the maximum \( \omega_z^* \) is only 55% larger than the maximum \( \omega_x^* \) or 47% larger than the maximum \( \omega_y^* \). The observation conforms to that of the rms values (Fig. 7) and reconfirms that the three-dimensionality of the flow is enhanced at higher \( Re \).

V. Coherent Contributions to Reynolds Stress and Spanwise Vorticity Variance

Assuming a phase-averaged structure beginning at \( k_1 \) samples (corresponding to \( \phi = -\pi \)) before \( \phi = 0 \) and ending at \( k_2 \) samples (corresponding to \( \phi = \pi \)) after \( \phi = 0 \), the structural average, denoted by a double overbar, is defined by

\[
\overline{\vec{\beta} \vec{y}} = \frac{1}{k_1 + k_2 + 1} \sum_{k=k_1}^{k_2} \vec{\beta} \vec{y} \tag{13}
\]

where \( \vec{\beta} \) and \( \vec{y} \) stand for vorticity components. The value of \( k_1 (= k_2) \) is 30, so that the duration \((k_1 + k_2 + 1)\) corresponds approximately to the average vortex-shedding period. The ratio of \( \overline{\vec{\beta} \vec{y}} \) to \( \overline{\vec{\beta} \vec{y}} \) provides a measure for the contribution from the coherent structures to the vorticity variance or Reynolds stresses.

As discussed earlier, the coherent component of \( \omega_x \) or \( \omega_y \) cannot be extracted at present. Therefore, only the coherent contribution to the spanwise vorticity variance is calculated. Figure 11 shows the cross-flow distribution of \( \overline{\omega_x \omega_z} \). Evidently, \( \overline{\omega_x \omega_z} \) decreases with an increase in \( Re \); the maximum \( \overline{\omega_x \omega_z} \) at \( x/d = 10 \) (Fig. 11a) is about 37% at \( Re = 2.5 \times 10^4 \), 32% at \( Re = 5.0 \times 10^4 \), and 21% at \( Re = 1.0 \times 10^5 \). The result is consistent with increasing Kelvin–Helmholtz vortices or three-dimensionality of the flow at higher \( Re \). A similar observation is made at \( x/d = 20 \) and 40 (Figs. 11b and 11c). But the coherent contribution to spanwise vorticity variance reduces rapidly for a larger \( x/d \) probably due to the break-up of spanwise structures.

VI. Conclusions

It has been observed that the Reynolds number has a significant influence on the three simultaneously measured components of the vorticity vector. The investigation leads to the following conclusions:

1) The instantaneous vorticity signals exhibit a significantly growing small-scale fluctuations for a higher \( Re \), especially from \( Re = 5.0 \times 10^4 \) to \( 1.0 \times 10^5 \). The \( Re \) range coincides with that previously reported of the evident generation of small-scale Kelvin–Helmholtz vortices based on PIV. Correspondingly, the rms values of streamwise and lateral vorticity components increase considerably. The observations indicate a rapidly enhanced three-dimensionality of the flow, which could be ascribed mostly to the effect of Kelvin–Helmholtz vortices.

2) The coherent vorticity field has been calculated based on the measured vorticity. At \( Re = 2.5 \times 10^4 \), the maximum \( \omega_z^* \) is twice as large as that of \( \omega_x^* \) or \( \omega_y^* \). However, at \( Re = 1.0 \times 10^5 \), the maximum \( \omega_z^* \) is merely 55% larger than the maximum \( \omega_x^* \) or 47% larger than the maximum \( \omega_y^* \). Correspondingly, the contribution from the coherent structures to spanwise vorticity variance declines considerably. The results suggest increased three-dimensionality of spanwise structures, which is consistent with the perception that the three-dimensionality of the flow is enhanced at higher \( Re \).

3) The maximum spanwise coherent vorticity is approximately independent of \( Re \), at least for the \( Re \) range presently investigated.

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