Sound radiation of orthogonally stiffened laminated composite plates under airborne and structure borne excitations

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1. Introduction

As laminated, fiber-reinforced composite structures are increasingly exploited in a wide range of engineering applications [1], stiffeners are commonly used to further reinforce the laminated composite structure. To deal with stiffened structures, a most straightforward way [2] is to equivalent the whole structure as an orthotropic uniform plate if the mechanical wavelengths are greater than stiffener spacing. To describe the coupling forces between the stiffeners and the base plate, the stiffeners are conveniently replaced with lumped mass and spring [3]. More accurately, Langley and Heron [4] have given general coupling matrix at plate/beam junction, which can be extended to predict reactive forces by stiffeners. To solve the governing equations, Mead and Pujara [3] developed the method of space harmonic expansion, which has been developed to investigate a variety of periodically stiffened structures [5–7]. Employing the technique of Fourier transform [8], Mace [9,10] investigated the response of plates with parallel and orthogonal stiffeners under fluid loading, whereas Takahashi [11] studied sound radiation from double-leaf structures with periodical connections. Both of the two approaches described above transform the governing equations into infinite sets of simultaneous algebraic equations and then truncate these into a finite range for numerical solutions.

In comparison with the isotropic material made structures, the laminated composite material made structure significantly increases the complexity of the theoretical modeling. To deal with the laminated composite structure, Yin et al. [12] extended Mace’s model [9] to unidirectional stiffened laminated composite plate based on the classical laminated composite plate theory (CLPT), in which the bending motion of metallic stiffeners has been accounted for. Also based on CLPT theory, Legault et al. [13] explored the effect of finite dimensions by comparing the spatially windowing periodic model with the Rayleigh–Ritz method. It is concluded that the periodic theory is inappropriate when the bending wavelength is smaller than the stiffener spacing. There also exists such a restriction in the present paper since the periodic theory is used here. Recently, a first order shear deformation theory (FSDT) is employed by Mejdi et al. [14] to consider the transverse shear strain of base plate, where the in plane motion of stiffeners has been taken into account. Whereas, the governing equations for stiffeners only apply to thin-walled isotropic beam (or uncoupled composite case) while not the general composite beam.

To develop a more accurate theoretical model, a layerwise shear deformable theory is applied to model the vibration of the laminate composite base plate, and the shear deformable beam theory is utilized to model the vibration of arbitrary thin-walled composite beam stiffeners. Note that the single-layer theories (e.g. CLPT or FSDT) used by previous researchers remain acceptable for thin bare plate, which probably induce significant deviations for thicker and stiffened plates. Different from the existing studies, numerical discussions specially focus on the flexural-extension and the flexural-torsion coupling effects caused by the material anisotropy of
composite base plate, as well as the flexural-torsion coupling effect due to the geometrical anisotropy of the stiffeners.

2. Mathematical formulation

With reference to Fig. 1, consider an infinite laminated composite plate reinforced by orthogonal line stiffeners (without plate like behavior) along the lines \( x = m_{l} \) and \( y = n_{l} \), with \( (m, n) \) representing integers and \((l_{x}, l_{y})\) denoting spacing respectively. The origin of the Cartesian coordinates is located at the junction of the orthogonal stiffeners. The structure is loaded by acoustic fluid on one side, i.e., the side without stiffeners. Under a layerwise shear deformable theory [15], the discrete laminated model can express the displacements for the \( i \)th layer of the composite plate as:

\[
\begin{align*}
\mathbf{u}^i(x, y, z, t) &= u^i_0(x, y, t) + z\phi^i_0(x, y, t) \\
\mathbf{v}^i(x, y, z, t) &= v^i_0(x, y, t) + z\phi^i_0(x, y, t) \\
\mathbf{w}^i(x, y, z, t) &= w^i_0(x, y, t)
\end{align*}
\]

(1)

where \((u^i_0, v^i_0, w^i_0)\) are the displacements of the plate along \((x, y, z)\) coordinate directions in the mid-plane of each layer, and \((\phi^i_0, \phi^i_1)\) denote the rotation displacements of the plate about the \((y, z)\) directions, respectively.

The Euler–Lagrange equations of the system incorporating the reaction forces due to the stiffeners may be written as:

\[
\begin{align*}
\frac{\partial N_{nx}}{\partial x} + \frac{\partial N_{ny}}{\partial y} - l_{0} \frac{\partial^{2} u_{0}^{i}}{\partial t^{2}} + l_{1} \frac{\partial^{2} \phi_{0}^{i}}{\partial t^{2}} + F_{x} - F_{x}^{i-1} &= f_{x}^{i} - \delta_{b} \left[ \sum_{m, z} F_{m} \delta(y - m l_{y}) + \sum_{n, z} F_{n} \delta(y - n l_{y}) \right] \\
\frac{\partial N_{ny}}{\partial y} + \frac{\partial N_{nx}}{\partial x} - l_{0} \frac{\partial^{2} v_{0}^{i}}{\partial t^{2}} + l_{1} \frac{\partial^{2} \phi_{0}^{i}}{\partial t^{2}} + F_{y} - F_{y}^{i-1} &= f_{y}^{i} - \delta_{b} \left[ \sum_{m, z} F_{m} \delta(y - m l_{y}) + \sum_{n, z} F_{n} \delta(y - n l_{y}) \right]
\end{align*}
\]

(2)\( (3)\)

where \(l_{0} = -z_{0} \phi_{0}^{i-1} - z_{0} \phi_{0}^{i+1}, v_{0} = z_{0} \phi_{0}^{i-1} + z_{0} \phi_{0}^{i+1}, w_{0} = w_{0}^{i-1} \) \((4)\)

In all, the \(5N + 3(N - 1)\) variables (Eq. (7)) correspond to \(5N\) dynamic equilibrium equations (Eqs. (2)–(6)) and \(3(N - 1)\) displacement continuity equations (Eq. (9)). Notice that the present layerwise shear deformable theory model can be degraded into the FSDT model if the number of total layers is set to be one.

To solve the governing equations, the method of Space Fourier Transformation (SFT) is employed here for its capability to consider the effects of fluid loading and infinite periodic structures, so that:

\[
\begin{align*}
\mathbf{\tilde{w}}(x, \beta) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{w}(x, y) e^{i(\pi x + \pi y) / \lambda} dx dy \\
\mathbf{u}(x, y) &= \frac{1}{i \pi^{2}} \frac{1}{\pi^{2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{\tilde{w}}(x, \beta) e^{-i(\pi x + \pi y) / \lambda} dxdy \beta
\end{align*}
\]

(10)\( (11)\)

Transforming Eqs. (2)–(6) and Eq. (9) then leads to:

\[
\langle A_{12} angle + i \langle A_{11} \rangle - \langle A_{0} \rangle (\mathbf{F}) = \langle \tilde{P}_{1} \rangle - \langle \tilde{P}_{0} \rangle - \langle \tilde{P}_{2} \rangle
\]

(12)

where \(i = \sqrt{-1}\), the coefficient matrices \(\{A_{0}, A_{1}, A_{2}\}\) are defined by Chinen and Atalla [15]. The coupling sound pressure \(P_{y}(x, \beta, 0) = -2c_{0}^{2} \rho_{0} \mathbf{w}(x, \beta) / \sqrt{2^{2} + \beta^{2} - \beta^{2} / c_{0}^{2}}\) \((c_{0}\) is the speed of sound, \(\rho_{0}\) is the density of fluid) at fluid–panel interface can be easily joined into the coefficient matrices. Then, the hybrid variables vector, excitation forces vector, reaction forces between the base plate and the stiffeners can be written as:

\[
\begin{align*}
\{\mathbf{e}\} &= \{\mathbf{\tilde{U}}^{T}, \mathbf{F}\}^{T}, \{\mathbf{P}_{1}\} = \{\mathbf{f}_{x}^{1,1}, \mathbf{f}_{x}^{1,0}, \mathbf{f}_{x}^{0,1}, \mathbf{f}_{x}^{0,0}, \mathbf{f}_{y}^{1,1}, \mathbf{f}_{y}^{1,0}, \mathbf{f}_{y}^{0,1}, \mathbf{f}_{y}^{0,0}, \mathbf{0}, \mathbf{0}\}^{T}, \\
\{\mathbf{P}_{2}\} &= \{\mathbf{F}_{y} \mathbf{F}_{x} \mathbf{F}_{z}, \mathbf{M}_{a}, \mathbf{0}, \mathbf{0}\}^{T}, \{\mathbf{P}_{3}\} = \{\mathbf{F}_{y}, \mathbf{F}_{x}, \mathbf{F}_{z}, \mathbf{0}, \mathbf{M}_{a}, \mathbf{0}, \mathbf{0}\}^{T}, \{\mathbf{P}_{4}\} = \{\mathbf{F}_{y}^{r}, \mathbf{F}_{x}^{r}, \mathbf{F}_{z}^{r}, \mathbf{0}, \mathbf{M}_{a}, \mathbf{0}, \mathbf{0}\}^{T}.
\end{align*}
\]

(13)

2.1. Reactive forces by stiffeners

As above mentioned in Section 1, the existing theoretical works about stiffened composite plate could only consider isotropic or uncoupled composite beam stiffeners. While, the shear deformable
beam theory that can handle arbitrary thin-walled composite beams is introduced here [16]:

\[
(Q) = \begin{bmatrix}
q_x & q_y & m_x & m_y \\
q_z & m_z & -z_c & \omega^* \frac{w}{y} x_c
\end{bmatrix} = [A](a)
\]

(14)

where \((Q), (a), [A]\) represent force vector, displacement vector and coefficient matrix, respectively. Remarkably, it is shown that various coupling effects may take place among these six unknown displacements variables. In the present paper, attention is paid to the flexural-torsion coupling effect due to the geometric rather than the material anisotropy of stiffeners. Then, the non-zero coefficients \(a_{ij}\) for C shape beam in \(y\)-direction are given:

\[
a_{11} = El_y \frac{d^2}{dx^2} - m_0 \omega^2; \quad a_{22} = -Ea \frac{d^2}{dx^2} - m_0 \omega^2; \quad a_{33} = El_y \frac{d^2}{dx^2} - Gj \frac{d^2}{dx^2} + (m_1 + m_2) \omega^2.
\]

(15)

where the definitions of all these constants can be found in Lee and Kim's paper [17]. All the damping effects in the present work are assumed to be structural damping and expressed in the form of complex Young's modulus. The relationship between the beam reaction forces \((|Q|)\) and base plate reaction forces \((|F|)\) can be written as [2]:

\[
(Q) = \begin{bmatrix}
q_x & q_y & m_x & m_y \\
q_z & m_z & -z_c & \omega^* \frac{w}{y} x_c
\end{bmatrix} = [R](F)
\]

(16)

where the point \((x_c, z_c)\) denotes the line junction between stiffeners and base plate with respect to shear center of stiffeners, \(\omega^*\) represents the warping function of stiffeners and \(\frac{w}{y}\) denotes the first-order derivative with respect to \(y\). Then, displacement continuity between the beam displacement \((a)\) and the base plate displacement \((b)\) requires:

\[
(b) = [u_1, v_1, w_1, \phi_1]^T = [R]^T(a)
\]

(17)

Notice that the flexural-torsion coupling effect of eccentric stiffeners has been reflected in the relation matrix \([R]\) explicitly. Combining Eqs. (15)–(17), the reactive forces of stiffeners are related to the base plate along the \(y\)-direction as:

\[
(F) = ([R]^T[A]^{-1}[R])^{-1}(b)
\]

(18)

To solve the Dirac function appearing in Eqs. (2)–(6), the Poisson formula is employed [10,11]:

\[
\sum_{m=2}^{\infty} \delta(x - ml_c) = \frac{1}{l_c} \sum_{m=2}^{\infty} \exp \left(\frac{2 \pi m \pi x}{l_c}\right)
\]

(19)

Upon incorporating Eqs. (18), (19) and introducing the definition \((x_m = x + 2m \pi \ell, \beta_m = \beta + 2m \pi \ell)\), the transform of the coupling forces between the stiffeners and the base plate along the \(y\)-direction is obtained as:

\[
\tilde{(F)}_y = \begin{bmatrix}
\tilde{F}_{xx} & \tilde{F}_{xy} & \tilde{F}_{xz} \\
\tilde{F}_{yy} & \tilde{F}_{yz} & \tilde{M}_y
\end{bmatrix} = \begin{bmatrix}
211 & 212 & 213 & 214 & 0 & \ldots \\
212 & 222 & 223 & 224 & 0 & \ldots \\
213 & 223 & 233 & 234 & 0 & \ldots \\
214 & 224 & 234 & 244 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(20)

where \(\tilde{Z}_w(i, j = 1 \sim 4)\) represents the transformed coefficient matrix elements. Similarly, the reactive forces by stiffeners along \(x\)-direction are expressed as:

\[
\tilde{(F)}_x = \begin{bmatrix}
\tilde{F}_{xx} & \tilde{F}_{xy} & \tilde{F}_{xz} \\
\tilde{F}_{yx} & \tilde{F}_{yy} & \tilde{F}_{yz} \\
\tilde{M}_x & \tilde{M}_y & \tilde{M}_z
\end{bmatrix} = \begin{bmatrix}
211 & 212 & 213 & 214 & 0 & \ldots \\
212 & 222 & 223 & 224 & 0 & \ldots \\
213 & 223 & 233 & 234 & 0 & \ldots \\
214 & 224 & 234 & 244 & 0 & \ldots \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(21)

Combining Eqs. (12), (20), and (21), the resultant governing equations in wavenumber domain can thence be given as:

\[
\begin{bmatrix}
\tilde{u}_1(x, \beta) \\
\tilde{v}_1(x, \beta) \\
\tilde{w}_1(x, \beta) \\
\tilde{\phi}_1(x, \beta)
\end{bmatrix} = \left[L_{5 \times 5}\right]^{-1}(\tilde{u}_1(x, \beta))
\]

(22)

where \(L_{5 \times 5}\) denotes the first five lines and columns of the inversion stiffness matrix in Eq. (12). To solve the coupling unknowns, two sets of intermediate variable are introduced here as:

\[
\begin{bmatrix}
\tilde{u}_1(x, \beta) \\
\tilde{v}_1(x, \beta) \\
\tilde{w}_1(x, \beta) \\
\tilde{\phi}_1(x, \beta)
\end{bmatrix} = \begin{bmatrix}
\sum_{n=2}^{\infty} \tilde{u}_1^n(x, \beta) \\
\sum_{n=2}^{\infty} \tilde{v}_1^n(x, \beta) \\
\sum_{n=2}^{\infty} \tilde{w}_1^n(x, \beta) \\
\sum_{n=2}^{\infty} \tilde{\phi}_1^n(x, \beta)
\end{bmatrix}^T
\]

(23)
Notice that the definition \((\xi(\alpha, \beta) = |\bar{u}(\alpha, \beta), \bar{p}(\alpha, \beta), \bar{w}(\alpha, \beta), \bar{w}(\alpha, \beta)|^2, x_m = \alpha + 2m\pi, \beta = \beta + 2n\pi)\), the intermediate variables have the periodicity properties:
\[
\xi_m(\alpha, \beta) = \xi_m(\alpha, \beta), \xi_m(\alpha, \beta) = \xi_m(\alpha, \beta)
\]

(24)

Once these intermediate variables of Eq. (23) are determined, the displacements in wavenumber domain can be obtained using Eq. (22). Given the definition of intermediate variables in Eq. (23) and their periodicity property shown in Eq. (24), summing Eq. (22) over all \(m\) or \(n\) values yields:
\[
\begin{align*}
\xi_m(\alpha, \beta) &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_{m}(\alpha, \beta) + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} S_{m}(\alpha, \beta) \xi_m(\alpha, \beta) + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} S_{m}(\alpha, \beta) \xi_m(\alpha, \beta) \\
\phi_m(\alpha, \beta) &= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_{m}(\alpha, \beta) + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} S_{m}(\alpha, \beta) \phi_m(\alpha, \beta) + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} S_{m}(\alpha, \beta) \phi_m(\alpha, \beta)
\end{align*}
\]

(25)

where \(P_m(\alpha, \beta), S_m(\alpha, \beta)\) and \(S_m(\alpha, \beta)\) correspond to the coefficient matrix in Eq. (22). Here, the sum-indices \((m, n)\) are restricted to have finite values, i.e., \(m = -m\) to \(m\) and \(n = -n\) to \(n\), thus Eq. (25) forms a system of \((2m + 1)\) linear equations for determining the intermediate variables \((\xi_m(\alpha, \beta), \phi_m(\alpha, \beta))\) (total \((2m + 1)) \) and \((\xi_m(\alpha, \beta), \phi_m(\alpha, \beta))\) (total \((2n + 1)) \). The convergence criteria said that once the solution was convergent at a given frequency, it is also convergent for all frequency lower than that \([18]\). Therefore, the convergence check is performed at the highest frequency 10 kHz of interest here, and it is found that the sum-indices taking values \(m = n = 15\) can ensure the convergence of the results within the error bound of 0.5 dB.

2.2. Far field radiated sound pressure and sound transmission loss

Following the standard procedure of stationary phase \([19]\), the far field acoustic radiation in spherical coordinates can be obtained if neglecting the radiation from stiffeners:
\[
P(R, \theta, \phi) = -p_0 Q \bar{w}(\theta, \phi) \exp(-ik_0 R)/2\pi R
\]

(26)

where \(p_0 = k_0 \sin \theta \cos \phi, \beta_0 = k_0 \sin \theta \sin \phi, p_0 = \) the density of fluid, \(k_0 = \omega / c_0\) is acoustic wavenumber, \((R, \theta, \phi)\) are the selected spherical coordinates. The high frequency asymptote of the far field sound pressure \(P = p_0 Q \exp (-ik_0 R)/2\pi R m(R)\) is the surface density of base plate, \(Q\) is the amplitude of imposed point force, radiation field by an unstiffened plate is selected here as a Ref. \([10]\). Then, the far field sound pressure can be expressed by sound pressure level (SPL = 20log10(P/P_s)) in decibel scales (dB).

Assuming both sides of the plate emerged in fluids and excited by sound pressure \(P\), the sound transmission problem can also be handled by the above formulated model based on the layerwise shear deformable theory, the incident sound intensity is defined as \(W_i = 1/(4\pi) \cos \phi / (\rho_0 c_0)\), where \(\phi\) is the sound incidence angle. The transmitted sound intensity is given in wavenumber domain as \([20]\)
\[
W_i = \frac{\rho_0 \cos^2}{8\pi^2} \sum_{(\alpha, \beta) \in (x_0, \beta_0), \xi}(\bar{w}(\alpha, \beta))^2 (27)
\]

Then, sound transmission loss (STL) is expressed by the formula \(\text{STL} = 10 \log_{10}(W_i/W_0)\). This theoretical model for predicting STL of infinite structure can also take account of finite size effect approximately by applying the windowing technique. Whereas, it will be cumbersome to execute the windowing process in wavenumber field following the classical spatial windowing method \([21]\). Alternatively, a finite radiation efficiency \(\sigma\) \([22]\) is adopted here to replace the infinite radiation efficiency in the calculation of the transmitted sound intensity, which can be written as:
\[
W_i = \frac{\rho_0 \cos^2}{8\pi^2} \sum_{(\alpha, \beta) \in (x_0, \beta_0), \xi}(\bar{w}(\alpha, \beta))^2
\]

(28)

3. Results and discussion

In this section, numerical calculations based on theoretical formulations presented above are performed to explore the vibro-acoustic characteristics of infinite laminated composite plates stiffened by orthogonal C shape stiffeners. Unless otherwise stated, the following material and geometry parameters are used as follows. The base plate is made of composite material with modulus \(E_1 = 150\) GPa, \(E_2 = 9.0\ GPa, G_{12} = G_{13} = 7.1\ GPa, G_{23} = 2.5\ GPa,\) density \(\rho = 1600\ \text{kg/m}^3,\) and thickness \(h = 0.012\ \text{m}.\) The acoustic fluid has a density \(\rho_0 = 1000\ \text{kg/m}^3\) with sound speed \(c_0 = 1500\ \text{m/s}.\) Parameters of C-shape stiffeners are chosen with modulus \(E = 195\ GPa,\) density \(\rho_1 = 7700\ \text{kg/m}^3,\) web width \(b_1 = 0.02\ \text{m},\) flange width \(b_2 = 0.02\ \text{m},\) thickness \(h_1 = 0.001\ \text{m},\) and stiffener spacing \(s = s = 0.2\ \text{m}.\)

3.1. Validation of theoretical modeling

To verify the validity of the present theoretical model, the predictions are compared with existing theoretical results of Mace \([10]\) for sound radiation of orthogonally stiffened uniform plates, as shown in Fig. 2. To indicate the advantage of the layerwise shear deformable theory based model, the predictions for the structures of the one layer and three layer configurations (with the same thickness) are also included in Fig. 2. Note that whilst Mace adopted the Kirchhoff thin plate theory to describe the base plate, the present model applying to the one layer configuration is actually degraded to the first order shear deformation theory (FSDT). The comparison of Fig. 2 demonstrates that the present results agree excellently well with Mace’s results over a wide frequency range. The discrepancies appearing approximately above 7000 Hz are mainly attributed to the fact that the torsion moments of the stiffeners are considered in the present theoretical model but not in Mace’s model, and another reason lies on the different plate theory. Moreover, it is interesting to note that, there exists significant difference between two different configurations for stiffened cases while no distinction is observed for unstiffened cases. This actually demonstrates the advantage of the present layerwise shear deformable theory based model, that is, it is necessary to adopt the more accurate theory (i.e., the layerwise shear deformable theory) to model the stiffened composite plates.

To further check the applicability of the present model, the infinite model predictions and the finite model predictions (i.e., the finite radiation efficiency is applied) are both compared with the published experimental results \([23]\) for the sound transmission
loss (1/3 octave) of unidirectional all metallic stiffened panel as shown in Fig. 3. In this experiment, the size of the 4 mm thick aluminum base panel is 2.73 m by 3.43 m, and the periodic aluminum stiffeners are spaced 40 mm apart. Again, the present model predictions are in good agreement with the experimental results especially in the coincidence region. The theoretical model with the finite radiation efficiency applied shows superiority over the infinite model in terms of the excellent agreements with experimental results over the whole frequency range, particularly in the low frequency region.

3.2. Flexural-extension coupling effect of composite bare plate

First, sound radiation from unstiffened laminated composite plates excited by a transverse point force is considered, which has been modeled by Yin and Cui [24] amongst others. To quantify the flexural-extension coupling effect, Fig. 4 presents the predicted sound pressure levels for two typical lamination schemes: symmetric [75/60/45] sym and anti-symmetric plies [75/60/45] antisym. The predictions by Yin and Cui [24] based on CLPT are also included for comparison.

Yin and Cui [24] demonstrated that the influence of different lamination schemes upon the SPL of a composite plate is negligible, as shown in Fig. 4. However, if the thickness of a single ply is increased from 1.5 mm to 2 mm (other parameters remain unchanged), there exists a noticeable discrepancy between the present predictions and Yin and Cui’s results especially in the high frequency regime. This actually implies that the layerwise shear deformable theory applied here is more accurate than the classical laminated plate theory (CLPT) adopted by Yin and Cui [24] for dealing with relatively thick composite plates for which the flexural-extension coupling effect can be well demonstrated. Also, the symmetric lamination scheme is found to produce stronger radiation pressure than the antisymmetric one. This is considered reasonable because flexural-extension coupling is absent in the symmetric scheme and hence less energy is converted from bending wave to longitudinal wave. Whereas, the coincidence of all four curves in the low frequency regime (<3000 Hz) of Fig. 4 should be attributed to the fact that the identical heavy fluid (i.e., water considered here) dominates the low-frequency dynamic response of composite plates [25].

To investigate further the flexural-extension coupling effect, Fig. 5 presents the radiated sound pressure level of symmetrical and antisymmetrical laminated schemes under a transverse point force excitation.

The predictions by Yin and Cui [24] based on CLPT are also included for comparison. The comparisons between present predictions and those of Yin and Cui [24] for symmetric and antisymmetric laminated schemes under a transverse point force excitation.

3.3. Flexural-torsion coupling effect of composite bare plate

The flexural-torsion coupling effect has been found to be important in designing forward-swept wing composite structures having enhanced aerodynamic performance [26]. How such coupling effect affects the vibroacoustic behaviors of a composite plate is therefore of significant interest but always unnoticed by previous researchers. To avoid the influence of other coupling effects, three symmetric schemes are selected, including single-layer configuration ([45°/45°/45°]), regular symmetric angle-ply configuration with three layers [45°/–45°/45°] and fifteen layers [45°/–45°/45°/–45°/45°/–45°/45°/–45°/45°/–45°/45°/–45°/45°]. The predictions are presented in Fig. 6. Notice that, the single layer configuration effect, the anti-symmetric plate can radiate much larger sound pressure compared to the symmetric one. Notice that, for in-plane force excitation, the flexural-extension coupling effect may be maximized because it is exactly this kind of coupling effect that converts longitudinal wave energy to bending wave energy. The results of Figs. 4 and 5 reveal that the SPL of a composite plate excited by unit in-plane force is much lower than that of a composite plate excited by unit transverse point force. This implies that the contribution from in-plane force excitation to radiation power may be neglected in comparison with that from transverse force excitation of the same amplitude.
means one scheme with flexural-torsion coupling effect, and this coupling effect decreases as the number of layers is increased for symmetric angle-ply configurations.

The results of Fig. 6 shows that the single layer configuration produces larger far-field pressure over the other two symmetric laminate schemes considered, which is consistent with existing results [27]. This is because the flexural-torsion coupling effect can enlarge the bending deformation under transverse loading. The observed discrepancy between single layer configuration and multi-layer configurations illustrates the necessity for considering the flexural-torsion coupling effect especially in the high frequency regime.

3.4. Flexural-torsion coupling effect of stiffeners

Whilst it has been well established that flexural-torsion coupling is important for beam structures with channel cross sections, it is yet unclear to what extent this coupling effect may affect the vibroacoustic response of a stiffened plate as such effect is usually neglected for simplicity [23]. Fig. 7 plots the predicted far field pressure of single layer \([45^\circ/-45^\circ/45^\circ\)] configuration excited by unit transverse point force, with and without considering the flexural-torsion coupling of the stiffeners. The results without flexural-torsion coupling are obtained by neglecting the torsion motion equations and coupling coefficient \(x_a\) in Eq. (15). As shown in Fig. 7, the flexural-torsion coupling effect of the stiffeners affects the tendency of the SPL curve of the structure mainly in the way of changing the peaks and dips in the SPL curve especially in higher frequency range. As a matter of fact, this proves the importance and necessity of the accurate stiffener modeling.

3.5. Influence of stiffener spacings

As the stiffener spacing \((l_x, l_y)\) is one of the most important geometric parameters governing wave propagation in the whole structure, it is of great significance to evaluate its influence on sound radiation characteristic of the structure. Fig. 8 illustrates the far field radiated sound pressure with \((l_x, l_y)\) selected as \((0.2, 0.2)\)m and \((0.25, 0.25)\)m respectively. The attractive phenomena on the results is that all the peaks and dips on the curve shift to lower frequency as the stiffener spacings are increased. In fact, the inter-stiffener panels can be treated as small bounded plates approximately as stated by Fahy and Gardonio [25], and the peaks or dips on the radiation curves correspond to resonance frequencies of these bounded plates. Therefore, larger spacing results in lower natural frequency for fixed mode order. Moreover, the approximate quantitative relation \(4215/2771 \approx (25/20)^2\) for two peaks as shown in the figure confirms the present explanation again.

4. Conclusions

In this study, a layerwise shear deformable theory is adopted to develop an analytical model for describing the vibroacoustic of orthogonally stiffened laminated composite plates, in which full account is given to various coupling effects including the flexural-extension and the flexural-torsion couplings of the composite base plates. Numerical results show that the coupling effects (i.e., the flexural-extension and the flexural-torsion couplings) owing to material anisotropic base plate and the geometrical anisotropic stiffeners can both play a significant influence on structure sound radiation, which are also influenced by specific material properties, lamination schemes and excitation frequencies etc. Particularly, the symmetry of the composite plate exerts a significant effect on the sound radiation of the considered structure when excited by an in-plane point force. Moreover, since the inter-stiffener panels can be treated as small bounded plates, the increase of the stiffener spacing leads to the decrease of the natural frequency of the structure for fixed mode order.
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