Sound absorption of microperforated panels inside compact acoustic enclosures

Cheng Yang, Li Cheng*

Department of Mechanical Engineering, The Hong Kong Polytechnic University, Hung Hom, Hong Kong Special Administrative Region

A R T I C L E  I N F O

Article history:
Received 22 January 2015
Received in revised form 16 September 2015
Accepted 16 September 2015
Handling Editor: R.E. Musaﬁr
Available online 3 October 2015

A B S T R A C T

This paper investigates the sound absorption effect of microperforated panels (MPPs) in small-scale enclosures, an effort stemming from the recent interests in using MPPs for noise control in compact mechanical systems. Two typical MPP backing cavity configurations (an empty backing cavity and a honeycomb backing structure) are studied. Although both configurations provide basically the same sound absorption curves from standard impedance tube measurements, their in situ sound absorption properties, when placed inside a small enclosure, are drastically different. This phenomenon is explained using a simple system model based on modal analyses. It is shown that the accurate prediction of the in situ sound absorption of the MPPs inside compact acoustic enclosures requires meticulous consideration of the configuration of the backing cavity and its coupling with the enclosure in front. The MPP structure should be treated as part of the entire system, rather than an absorption boundary characterized by the surface impedance, calculated or measured in simple acoustic environment. Considering the spatial matching between the acoustic ﬁelds across the MPP, the possibility of attenuating particular enclosure resonances by partially covering the enclosure wall with a properly designed MPP structure is also demonstrated.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

Microperforated panels (MPPs) exhibit appealing features as compared with conventional fibrous sound-absorbing materials. The acoustic impedance of an MPP can be predicted by the formula proposed by Maa [1], as an extension of the short tube theory of Rayleigh and Crandall. In order to achieve effective sound absorption, an air layer is usually required to be introduced between the MPP and the backing rigid wall to generate the Helmholtz resonance effect. This typical MPP structure is usually referred to as the microperforated panel absorber (MPPA) [1]. Extensive efforts were made to improve the sound absorption of various MPP configurations, as reported in the open literature [2–12].

Earlier, the MPPs were used in architectural acoustics and for controlling environmental noise, where simple acoustic ﬁeld is usually considered, either diffuse or plane wave in most cases. An MPPA, in this connection, is usually treated as a sound-absorbing material with its acoustic properties characterized by the surface impedance or sound absorption coefﬁcient, measured in either a reverberation chamber or a Kundt’s tube. For example, in a large room, sound-absorbing materials, when attached to the wall of an enclosure, reduce the sound reﬂection through energy dissipation. Assuming a
sufficiently high modal density of the enclosure, the acoustic field may be considered as diffuse and the enclosure boundaries, where the sound-absorbing materials are placed, could be treated as locally reactive, showing that the response at one point is independent of the response at any other point [13,14], which applies for most music halls and industrial rooms within the audio frequency range. This treatment is generally well accepted, although some experimental work showed that the acoustic field inside a reverberation room with a flexible wall on the boundary cannot be correctly estimated in terms of the locally reactive normal acoustic impedance in the low-frequency range [15].

The unique physical property of MPPAs also shows a great potential for noise control in complex mechanical systems such as magnetic resonance imaging (MRI) scanners [16], cooling systems [17], nacelles of turbofan engines [18], and interiors of engine enclosures [19]. A common feature of these problems is that the MPPA may closely interact with the surrounding vibroacoustic elements, thus creating problems that are not encountered by typical architectural and environmental acoustics. This calls for revisiting and interrogating the in situ sound absorption mechanism of the MPPA under more complex vibroacoustic working environment by considering its interaction with the surrounding acoustic media.

The complex vibroacoustic behaviors of the MPPA also attracted the attention of researchers. One example is the observation of unexpected peaks in the sound absorption curve attributed to the structural resonance of the MPP [20]. Simultaneously, modal expansion methods were also used to investigate the vibroacoustic behaviors of the MPPA [21,22], shedding light on the structural effect of MPPs on the absorption performance. In addition to the structural vibration, the sound absorption behavior is also dependent on the acoustic field inside the MPPA backing cavity. For example, appreciable changes were observed in the sound absorption curve when the wall of the backing cavity is slightly tilted [23]. In addition to the vibroacoustic property of the MPPA itself, the acoustic field in front also affects the sound absorption performance. For example, a previous work [24] demonstrated that when an MPPA is subjected to an oblique plane wave, it behaves differently at different incident angles, because of the dominant contribution of different types of acoustic modes to the backing cavity acoustic field.

Despite the progressive efforts made in the past, the in situ sound absorption behavior of MPPAs in a strongly coupled vibroacoustic system has never been systematically documented in the literature, to the best of the authors’ knowledge. More importantly, the influence of the surrounding acoustic fields on the MPP, the extent to which they affect the energy dissipation inside the MPP pores, and the possibility of designing a suitable MPPA to suppress particular system resonances are the important areas to be explored. As a continuation of the previous work [24], this study investigates the aforementioned issues by emphasizing the application of MPPs in compact acoustic enclosures. By virtue of the remoteness of acoustic modes in the frequency domain, the acoustic fields of the so-called compact enclosures exhibit distinguishable modal features, and the absorption boundary corresponding to MPPAs may have strong interactions with the enclosure. This study also underscores the importance of considering MPPA as part of the entire acoustic system rather than as an absorption boundary characterized by the locally reactive impedance. As part of an acoustic system, the performance of the MPPA would be strongly influenced by the surrounding acoustic media to which it is coupled.

This article is organized as follows: Section 2 reviews the development of the acoustic impedance of the MPPA. In Section 3, a fully coupled model for an MPPA coupled with an enclosure is established using modal expansion method. In Section 4, an experiment is conducted to show the in situ sound absorption of MPPAs with two different backing cavity configurations, one with an empty backing cavity and the other with a honeycomb backing structure, inside a rectangular enclosure. The underlying physics, observed in the experiment, is numerically studied and experimentally validated in Section 5. The partial coverage of MPPA on the cavity wall is then investigated, demonstrating the possibility of designing an MPPA backing cavity as well as its location to cope with particular cavity resonances.

2. Locally reactive impedance formula

Maa’s locally reactive model is briefed first for the sake of completeness. Fig. 1 depicts an MPPA subjected to normal plane wave incidence, which consists of an MPP and a rigid wall, separated by an air layer of depth $D$.

![Fig. 1. A typical locally reactive model for the MPP construction. The backing wall (bottom line) of the air layer is rigid.](image-url)
If the separation between the pores is large enough compared with the pore diameters, the acoustic impedance of the panel, $Z_{\text{MPP}}$, can be approximated as the quotient of acoustic impedance of the individual pore and the perforation ratio (ratio of areas of pores to panel) [1].

For plane wave, the relationship between the acoustic impedances of the two boundaries of the air layer is given by [25]

$$
Z(0) = \frac{Z(D) + j\rho c \tan(kD)}{1 + j\rho c \tan(kD)Z(D)}
$$

(1)

where $\rho$ and $c$ are the air density and sound speed, respectively. $k = \omega/c$ is the wavenumber, with $\omega$ being the angular frequency. With a rigid backing wall, the acoustic impedance (or the reactance, because this term is purely imaginary) at the top of the air layer becomes

$$
Z(0) = Z_{\text{Cav}} = -j\rho c \cot(kD).
$$

(2)

According to the equivalent electro-acoustic approach, the acoustic impedances of the MPP and the backing air layer are arranged in series. Therefore, the total acoustic impedance of the MPPA is

$$
Z_{\text{MPPA}} = Z_{\text{MPP}} + Z_{\text{Cav}} = Z_{\text{MPP}} - j\rho c \cot(kD).
$$

(3)

Eq. (3) suggests that the air mass inside the pores vibrates independently in the $x$ direction, which is normal to the panel surface, and the reactance offered by the air layer is uniform across the MPP surface. Therefore, the MPPA modeled in this manner is locally reactive. Fig. 2 depicts the magnitude of the imaginary part of $Z_{\text{MPPA}}$ in terms of its two components: $Z_{\text{MPP}}$ and $Z_{\text{Cav}}$. It can be seen that, once the reactance of $Z_{\text{MPP}}$ intersects with the negative part of the reactance of the backing air layer (solid line), a sound absorption peak appears (circle). At these frequencies, the total reactance vanishes and the MPPA works as a Helmholtz resonator at its resonant frequencies. However, the frequencies at which the reactance magnitude of the backing air layer is extremely large correspond to the absorption dips (triangle). Between each pair of peak and dip, where the variation of the reactance of the backing air layer is moderate, good sound absorption is obtained. The reactance magnitude of the backing cavity is quite large at low frequencies, restricting the use of MPPA in low-frequency noise unless a large cavity depth $D$ is deployed. Recent studies [26,27] attempt to overcome this problem using active control techniques to reduce larger reactance of the backing air layer, yielding higher sound absorption coefficient in the low-frequency range.

Maa also developed a model for the oblique incidence case, in which the reactance provided by the backing air layer is a function of the wave incidence angle and is dependent on the path difference between the incident and reflected waves. However, that model could not be directly used as an impedance boundary in an enclosure unless a prior knowledge on the incidence angle at each frequency is available. Up to now, it has only been used to calculate the absorption coefficient in diffuse field for architectural acoustic problems to which the sound decay rate of an enclosure is related. It is also worth noting that, when the backing air layer is laterally bounded and has a finite lateral size, the whole assembly becomes nonlocally reactive [28]. In that case, the locally reactive model cannot fully describe the acoustic behavior of the MPPA.

---

**Fig. 2.** (a) Magnitude of the reactance of the MPP construction. Negative $Z_{\text{Cav}}$: , positive $Z_{\text{Cav}}$: , $Z_{\text{MPP}}$: . (b) Corresponding sound absorption curve of the MPPA.
3. A fully coupled enclosure – MPPA model

A model for an acoustic enclosure coupled with an MPPA is developed based on classical modal method, which has been widely used to study the vibroacoustic coupling problems in different configurations [29–31]. Assuming that the MPPA is nonlocally reactive, the acoustic field in the backing cavity is modeled as part of the system.

Fig. 3 shows an enclosure having an MPPA flush-mounted on one of its walls. Note that the MPP coverage can be either full or partial over the enclosure wall. The enclosure (domain 1) has physical boundaries comprising acoustically rigid walls and the MPP, excited by an acoustic point source, $Q(r_s)$. The backing cavity of the MPPA is modeled as another acoustic domain (domain 2).

Once activated, the motion of air inside the MPP pores becomes a secondary source, radiating sound into domains 1 and 2 simultaneously. In a harmonic regime, the acoustic pressure field in domain 1 can be described by the Kirchhoff–Helmholtz integral equation as [32]

$$ p_1 = -j \rho \omega \int_{S_a} G_1 v_1 \, dS_a + \int_{V_a} G_1 Q \, dV_s, $$

where $G_1$ is the Green’s function for domain 1, $v_1$ is the averaged normal air particle velocity over the MPP surface $S_a$ (positive outward), and $Q(r_s) = j \rho \omega q \delta(r - r_s)$, with $q$ being the volume velocity of source and $\delta(r)$ the Dirac delta function.

Eq. (4) indicates that the overall acoustic pressure field in domain 1 is composed of two parts: (1) boundary radiation due to the normal velocity of the MPP surface and (2) boundary radiation by the point source with all boundaries being acoustically rigid.

The acoustic field in domain 2 is only determined by the velocity of the MPP surface, which can be expressed as

$$ p_2 = -j \rho \omega \int_{S_a} G_2 v_2 \, dS_a. $$

The motion of the air particle over the MPP surface is a result of the pressure difference across its surface, described as

$$ v_1 = \frac{p_1 - p_2}{Z_{\text{MPP}}}. $$

Given a very thin MPP, the velocity of the air particles is assumed to have the same magnitude across the pores:

$$ v_1 = -v_2. $$

The structural vibration of panel is ignored for simplicity.

The acoustic pressure in domains 1 and 2 are expanded in terms of their respective rigid-walled cavity modes, $\phi_m$ and $\psi_n$ as

$$ p_1 = \sum_m A_m \phi_m $$

$$ p_2 = \sum_n B_n \psi_n. $$

Meanwhile, Green’s function in each cavity, which satisfies the Neumann boundary condition, can also be obtained by normal modal expansion in terms of its rigid-walled modes as

$$ G_1(r, r') = \sum_m \frac{\phi_m(r) \phi_m(r')} {A_m (k^2_m - k^2)} $$

![Fig. 3. MPPA coupled to the enclosure.](image)
Green’s functions are then introduced into Eqs. (4) and (5) to calculate the acoustic pressure of each domain. Applying the boundary conditions, Eqs. (6) and (7) yield

\[ p_1 = -j\rho \omega \int_{S_a} G_1 \frac{p_1 - p_2}{Z_{\text{MPP}}} dS_a + \int_{V_1} G_1 Q dV_s \]  

\[ p_2 = -j\rho \omega \int_{S_a} G_2 \frac{p_2 - p_1}{Z_{\text{MPP}}} dS_a. \]  

where \( k_{1m} \) and \( k_{2n} \) are the wavenumbers for the modes \( m \) and \( n \) in domains 1 and 2, respectively.

Fig. 4. Experimental setup: (a) enclosure; (b) MPPA having an empty air cavity; and (c) MPPA having a honeycomb structure inside the backing cavity.
Then, substituting Eqs. (8)–(13) into Eqs. (14) and (15) and using the orthogonal property, one obtains

\[
\left( k^2_1 - k^2 \right) A_{1m} A_m + j k C_{MPP} \sum_n r_{m,n}^1 A_m - j k C_{MPP} \sum_n R_{m,n} B_n = j \rho c k q \phi_m \phi_n (r_x)
\]

(16)

\[
\left( k^2_2 - k^2 \right) A_{2n} B_n + j k C_{MPP} \sum_n t_{n,n}^{(2)} B_n - j k C_{MPP} \sum_m R_{m,n} A_m = 0.
\]

(17)

where \( C_{MPP} = \rho c / Z_{MPP} \) is the specific acoustic admittance of the MPP.

Obviously, the cavity modes of the original enclosure are modified by the MPPA, the influence of which is manifested as the specific acoustic modal admittance of the MPP weighted by the auto- and cross-modal coupling coefficients defined as

\[
t_{m,m}^{(1)} = \int S \phi_m \phi_n \, dS_n
\]

(18)

\[
t_{n,n}^{(2)} = \int S \psi_n \psi_n \, dS_n
\]

(19)

\[
R_{m,n} = \int S \phi_m \psi_n \, dS_n
\]

(20)

In total, there are three groups of modal coupling coefficient in the above definitions. The first two apply to each domain and are the results of the MPP serving as an impedance boundary. The last one describes the modal interaction between the two acoustic domains, as a result of the velocity continuity between the two sides of the MPP, analogous to the connection of two acoustic cavities with a virtual panel as the interface [25], through which the backing cavity is coupled with the enclosure.

The coupled system formulated in Eqs. (16) and (17) can be written as a \((M+N)\) matrix equation, where \( M \) and \( N \) are the number of modes of the enclosure and that of the MPPA backing cavity under consideration, respectively. It can be solved by standard method upon a proper modal truncation. In addition, the model takes into account the acoustic coupling between the two domains \((1 \text{ and } 2)\) through the modal coupling coefficients defined in Eq. (20). This feature essentially results in the difference between the locally reactive model and the proposed model, to be demonstrated in the following sections.

4. Experimental observations

First, a right parallelepiped enclosure, shown in Fig. 4(a), was tested experimentally. The enclosure was fabricated using a 30-mm-thick acrylic panel. The inner dimensions of the enclosure are listed in Table 1.

Two MPPAs with different backing configurations were studied: one has an entire air volume and the other has a honeycomb structure at the back of the MPP. The former, shown in Fig. 4(b), was installed on the enclosure by replacing the original wall of the enclosure at \( y = 0.63 \) m. Parameters of the MPP and the dimensions of the backing cavity are also tabulated in Table 1. For the latter, the backing honeycomb structure is shown in Fig. 4(a) and (c).

In the experiment, a loudspeaker was used to excite the enclosure. The loudspeaker was mounted outside the cavity wall, feeding acoustic excitation to the enclosure through an acrylic cone at \((0.06, 0, 0.06)\). The acoustic excitation strength is quantified using the transfer function between the two Bruel & Kjær 4942″ microphones, located at \((0.291, 0.547, 0.175)\) and \((0.06, 0.015, 0.06)\), respectively. The location of the first microphone (observation) was randomly selected, and the second microphone (reference) was placed very close to the apex of the cone. The transfer function (TF) between the two microphones is defined as

\[
\text{TF} = 20 \log_{10} \frac{P_{\text{mic1}}}{P_{\text{mic2}}}
\]

(21)

4.1. Enclosure without MPPA

Validation of the experimental model was first performed before mounting the MPPA to provide a benchmark for further evaluations. The predicted and measured TFs are compared in Fig. 5. In the simulation, a loss factor of 0.001 is used. As the

<p>| Table 1 |</p>
<table>
<thead>
<tr>
<th>Dimensions and parameters of the enclosure and MPPA used in the experimental investigation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enclosure</td>
</tr>
<tr>
<td>Lx</td>
</tr>
<tr>
<td>0.38 m</td>
</tr>
</tbody>
</table>
result shows, a frequency shift is found at a very low frequency of around 50 Hz, which may be caused by the opening that holds the cone apex [33]. Discrepancies at higher frequencies are possibly due to the acoustic scattering on the surfaces of cables and microphones. In general, the proposed model agrees well with the experimental results. The loss factor used in the simulation seems to be an adequate estimation of the system damping. Thus, the platform offers a convincing baseline for further investigations.

4.2. Sound absorption effect of the MPPAs

The in situ sound absorption of the two MPPAs is investigated. The measured TFs (Eq. (21)), corresponding to three different configurations (without MPPA, with MPPA having an empty backing cavity, and with MPPA having a honeycomb-filled backing cavity), are plotted and compared in Fig. 6. As a reference, the sound absorption coefficient curves of the two MPPAs measured from the impedance tube tests are also given in Fig. 7. It can be seen that, although the two MPPAs present very similar sound absorption curves (Fig. 7), their in situ sound absorption performances, as shown in Fig. 6, are quite different. This difference can be observed over a broad frequency band. In addition to the obvious superiority of the honeycomb backing configuration over the empty one, a closer examination reveals the deficiency of the latter. More specifically, in the frequency range where high sound absorption is expected from the impedance tube test (500–1200 Hz), the MPPA with an empty backing cavity fails to render the expected noise reduction at many dominant frequencies. These phenomena will be thoroughly investigated in the following sections.
5. Analyses

5.1. MPPA with an entire air volume at the back

In order to explain the aforementioned phenomena, an MPPA with an entire empty air volume at the back is investigated first. Numerical analyses are carried out in two-dimensional space perpendicular to the MPP surface for convenience. Parameters of the enclosure and the MPPA are listed in Table 2. A point source having a unit volume velocity is assumed around the corner at (0.03, 0.03).

For analyses, a space-averaged quadratic sound pressure inside the enclosure is defined as

\[ \langle p_1 p_1' \rangle = \frac{1}{V_1} \sum m |A_m|^2 A_{1m}. \tag{22} \]

The enclosure–MPPA coupled model is used for the analyses. In parallel, the model based on standard boundary integral method that treats the MPPA as an impedance boundary is also used for comparison. In the latter case, the effect of the MPPA backing cavity is embedded in the impedance formula, so that the modal interaction between the domains 1 and 2 is omitted and the coupled Eqs. (16) and (17) retreat to one equation as follows:

\[ (k^2_{1m} - k^2) A_{1m} A_m + jk C_{\text{MPPA}} \sum_{m'} \tilde{L}_{1m,m'} A_{m'} = j \rho c k q \phi m(t_s). \tag{23} \]

where \( C_{\text{MPPA}} = \rho c / Z_{\text{MPPA}} \) is the specific acoustic admittance of MPPA with \( Z_{\text{MPPA}} \) defined in Eq. (3). This model is referred to as the locally reactive model as opposed to the fully coupled model described in Section 3.

Fig. 8 shows the plot of the space-averaged quadratic sound pressures against frequency for three cases: enclosure without MPPA (with all acoustically rigid boundaries), enclosure with MPPA with locally reactive model, and enclosure with MPPA with fully coupled model. It can be seen that the locally reactive model predicts a broadband sound absorption with frequency ranging roughly from 300 to 1600 Hz, in agreement with the sound absorption coefficient curve obtained in impedance tube (not shown). Almost all the resonances of the rigid-walled cavity are damped due to the modal damping introduced by the MPPA. Besides, frequency shifts are observed at certain peaks attributed to the reactance term of the boundary impedance. However, for its counterpart, that is, the fully coupled model, MPPA fails to suppress several resonances as denoted by arrows in the figure, in agreement with the experimental observations reported in Section 4.2. These peaks are neither damped nor shifted, and have approximately the same magnitudes as those for the enclosure without MPPA. Obviously, the locally reactive model overestimates the sound absorption effect of the MPPA with an empty backing cavity.

---

**Table 2**
Dimensions and parameters of the enclosure and MPPA used in numerical analysis. (following the schematic in Fig. 3).

<table>
<thead>
<tr>
<th>Enclosure</th>
<th>MPPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lx</td>
<td>Lx</td>
</tr>
<tr>
<td>Ly</td>
<td>Ly</td>
</tr>
<tr>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>Pore diameter</td>
<td>Diam.</td>
</tr>
<tr>
<td>Panel thickness</td>
<td>Panel thickness</td>
</tr>
<tr>
<td>Perforation</td>
<td>Perforation</td>
</tr>
<tr>
<td>1 m</td>
<td>0.4 m</td>
</tr>
<tr>
<td>0.05 m</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>0.2 mm</td>
<td>1%</td>
</tr>
</tbody>
</table>
In the fully coupled model, MPP couples the enclosure and the MPPA backing cavity by its surface, through which the original modes of the rigid-walled enclosure are modified. The extent to which this modification occurs is quantified by measuring the wave matching between the modes in the two subsystems [35]. At this point, it is worthwhile to study the modal property of the coupled system to provide a physical explanation for the aforementioned phenomena. Following an iterative calculation scheme, the first few eigenvalues corresponding to the modes of the coupled system (enclosure+MPP+backing cavity), normalized by $c/(2Lx)$, are plotted in Fig. 9. These modes are termed as new modes in order to distinguish them from the original modes of the uncoupled systems. Each eigenvalue is a complex number, whose real and imaginary parts are associated with the generalized natural frequency and the loss factor of a new mode, respectively. Fig. 9 shows two distinct groups of new modes: one has eigenvalues with appreciable imaginary parts and the other with negligible imaginary parts. The former group corresponds to the damped new modes because of the energy dissipation of the MPP, whereas the latter group corresponds to the undamped new modes. More specifically, the real parts of those undamped new modes are all integers (1–5 in Fig. 9), having the same resonant frequencies as those of the lateral modes (vibrating in the direction parallel to the MPP) of the uncoupled enclosure.

Upon solving the eigenvalues of the coupled system, eigenvectors are obtained. The components of each eigenvector quantify the contributions of the modes of the subsystems in constructing the corresponding new mode. As an example, the magnitudes of the normalized modal coefficients, which contribute to the fourth undamped new mode, are plotted in Fig. 10 (a). It shows that this new mode is predominantly and equally contributed by two lateral modes: the (4, 0) mode of enclosure and the (4, 0) mode of the MPPA backing cavity, each of which describes the acoustic pressure distribution of the new mode in the corresponding domain (Note that (X, 0) modes involve acoustic pressure variations along the MPP surface.). Further examination shows that the two dominant modes also have the same phase (not shown). The in-phase vibration of the two dominant modes with identical modal amplitudes, arising from the strong coupling due to the perfect
wave matching of the dominating modes with the same resonant frequencies, yields zero pressure difference across the
MPP. Thus, there is no air motion inside the MPP pores, and no energy can possibly be dissipated. In such circumstances, the
MPP is analogous to a rigid panel. Recalling the result in Fig. 8, these undamped resonances correspond to the resonant
frequencies of the rigid-walled modes of the uncoupled enclosure indexed by (1, 0)–(9, 0).

The analyses described above provide an explanation to the ineffectiveness of the MPPA at some resonances (Fig. 8). The
phenomenon is attributed to the geometric similarity between the enclosure and the MPPA backing cavity in the x direction
that holds the same lateral modes. If the new modes are mainly dominated by the depth modes (vibrate in the direction
perpendicular to the MPP) of the subsystems, MPP may be activated. As an example, the magnitudes of the normalized
modal coefficients of the contributing modes for a damped new mode (with a generalized eigenvalue 4.4 + 0.3j) are plotted
in Fig. 10(b). Obviously, the modal behavior is mainly attributed to the depth modes and the volume mode, leading to a
considerable loss factor.

In the locally reactive impedance formula for the MPP with a backing layer, the acoustic wave is assumed to only propa-
gate in the direction normal to the MPP surface. Upon this assumption, the standing wave modes formed between the
lateral boundaries of a finitely bounded MPPA are disregarded, and only the depth modes of the backing cavity are con-
sidered. As a comparison, the generalized eigenvalues for the enclosure with the MPPA modeled as locally reactive impe-
dance boundary are also solved and plotted (Fig. 9). It can be seen that the model predicts cross-border modal damping
factors for almost all modes, thus resulting in an overestimation of the sound attenuation within the enclosure, in agree-
ment with the observations from Fig. 8.

The involvement of the lateral modes in constructing the acoustic field of the backing cavity also depends on the nature
of the acoustic media to which the MPPA is coupled. For instance, the lateral modes cannot be activated when an MPP is
subjected to a normal plane wave incidence, because of the mismatching of the waves on the two sides of the MPP. In other
words, if the acoustic pressure loading on the MPPA surface is uniform across the MPP surface, for example, plane wave

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig10.png}
\caption{Normalized modal coefficients (eigenvector) for the corresponding generalized eigenvalue: (a) 4 + 0j (undamped new mode); and (b) 4.4 + 0.3j (damped new mode).}
\end{figure}
incidence in impedance tube, only the depth modes of the backing cavity would be activated. Therefore, locally reactive model could be used only in the absence of lateral modes. Otherwise, as both numerically and experimentally observed, locally reactive model based on impedance tube measurement cannot truthfully reflect the in situ sound absorption capability of the MPPA in a compact vibroacoustic environment. A fully coupled model becomes indispensable in that case.

In order to further consolidate the above analyses, the predicted and measured TFs are compared in Fig. 11 using the three-dimensional configuration presented in Section 4, showing a good agreement between the two curves. This confirms that the acoustic field in the backing air cavity has to be carefully addressed and the proposed coupled model is reliable to achieve a fairly accurate prediction. In addition, the possible structural vibration of the MPP seems to have no significant influence on the acoustic field in the experiment. Despite the exclusion of the panel vibration from the proposed model, it is still acceptably accurate. Then, the measured TFs in the enclosure with and without MPPA are compared in Fig. 12, to further confirm the existence of the lateral modes in the MPPA backing cavity and their effective roles in the sound absorption performance. As marked in the figure, some resonances are unchanged after installing the MPPA. According to the modal indices listed in the figure, these resonances are associated with the lateral modes of the enclosure, in agreement with the aforementioned observations and conclusions.

5.2. MPPA with honeycomb structure at the back

In practice, an MPPA usually has a shallow backing cavity for space-saving purpose. Consequently, the resonant frequencies of the lateral modes may easily fall within the frequency range of interest. One way to avoid the formation of the lateral modes is to fill up the backing cavity with a honeycomb core, by which the cut-off frequency of the original backing cavity is greatly increased. Below the cut-off frequency, the acoustic wave inside each honeycomb cell is planar and can be
considered as propagating in the direction normal to the MPP surface only. Meanwhile, the honeycomb structure also helps in enhancing the strength of the MPPA.

The previously proposed fully coupled model can be revised to model such a honeycomb-backed MPPA. Following the major steps in Section 3, when a honeycomb structure is placed within the backing cavity, the original air volume is partitioned into a series of small subcavities corresponding to honeycomb cells. For the $i$th honeycomb cell, its acoustic field can be expressed as

$$p_i = -j\rho \omega \int_{S_i} G_i v_2(r_i) dS_i,$$  \hspace{1cm} (24)

where $S_i$ and $G_i$ are the cross-sectional area and the Green’s function of the $i$th honeycomb cell, and $v_2(r_i)$ is the normal velocity on MPP surface in front of the $i$th honeycomb cell.

The acoustic pressure in domain 1 can be expressed by the sum of the total acoustic radiation by each cell as follows:

$$p_1 = -j\rho \omega \sum_i \int_{S_i} G_i v_1(r_i) dS_i + \int_{V_i} G_1 Q dV_s.$$  \hspace{1cm} (25)

Expanding the acoustic pressure of the $i$th honeycomb cell yields

$$p_i = \sum_n B_{ni} \psi_n.$$  \hspace{1cm} (26)

The equations describing the enclosure coupled with the MPPA containing the honeycomb-backing structure then become

$$(k_m^2 - k^2) A_m + jkC_{MPP} \sum_i \left( \sum_n L_{mn,m}^{(1, i)} A_n - \sum_n R_{mn,n}^{(0)} B_{ni} \right) = j\rho c k q \phi_m(r_s)$$  \hspace{1cm} (27)

$$\left( k_n^2 - k^2 \right) A_{2n,i} + jkC_{MPP} \sum_m l_n^{(1, i)} B_{ni} - jkC_{MPP} \sum_m r_n^{(1, i)} A_m = 0$$  \hspace{1cm} (28)

for the $i$th honeycomb cell, where the modal coupling coefficients are

$$L_{mn,m}^{(1, i)} = \int_{S_i} \phi_m \phi_m dS_i$$  \hspace{1cm} (29)

$$l_n^{(1, i)} = \int_{S_i} \psi_n \psi_n dS_i$$  \hspace{1cm} (30)

$$R_{mn,n}^{(0)} = \int_{S_i} \phi_m \psi_n dS_i.$$  \hspace{1cm} (31)

Eqs. (27) and (28) can be written in the form of a $(M+N \times I)$ matrix form, where $I$ is the number of honeycomb cells. Fig. 13 compares the measured and predicted TFs. Predictions are made using the model described above and the locally reactive model formulated by Eq. (23). The agreement between the experimental and numerical results using both models seems to be very satisfactory. Most importantly, both models also agree well with each other. This is understandable, because with a honeycomb structure, the acoustic field in each backing cell is not directly coupled, but indirectly through

![Fig. 13. Transfer functions (TFs) for the enclosure coupled with a MPPA having honeycomb structure inside the backing cavity.](image-url)
the enclosure. Given that the dimensions of the cells are small, this indirect coupling is rather weak so that the MPPA behaves mainly in a locally reactive manner. In this case, an MPPA could be treated roughly as an impedance boundary using its locally reactive normal acoustic impedance. This simplification greatly reduces the computation time compared with the fully coupled model. However, it is important to note that such simplification holds only when the cross-sectional area of the honeycomb cells is comparably smaller than the wavelength of interest so that the contributions of the lateral modes inside the cells are negligible, the rule of thumb being less than a quarter of the smallest wavelength [36]. Otherwise, lateral modes should be considered, requiring the use of the coupled model presented in this article.

5.3. Enclosure partially covered by MPPA

The analyses described above consider full MPPA coverage on one of the enclosure walls, giving rise to the same lateral modes because of the geometric similarity between the two cavities. As shown both numerically and experimentally, an MPPA containing an entire backing volume shows deficiencies because of the perfect wave matching at the resonant frequencies of the lateral modes. Meanwhile, this may also suggest the possibility of using MPPA backing cavity, particularly its coupling with the front acoustic field, to cope with particular enclosure resonances. This issue is examined by investigating partial MPPA coverage with varying locations on the enclosure wall.

The system shown in Fig. 3 is considered again, with an entire air volume-backed MPPA, flush-mounted partially on the top wall of the enclosure from \((W, 0.4)\) to \((W+0.28, 0.4)\), where the width of the MPPA is fixed at an arbitrarily chosen value of 0.28 m. The value of \(W\) varies, thereby changing the MPPA location. The observed dimensions of the enclosure, the depth of the air gap behind the MPP, and the parameters of the MPP are similar to those listed in Table 2. Without loss of generality, a resonance at 1548 Hz corresponding to the \((9, 0)\) mode of the enclosure is arbitrarily chosen as the targeted frequency to be controlled. The width of the MPPA is chosen in such a way that the resonant frequency of the targeted enclosure mode falls within the interval between the resonant frequencies of the \((2, 0)\) and \((3, 0)\) modes of MPPA backing cavity.

The space-averaged quadratic sound pressure inside the enclosure with and without MPPA is depicted in Fig. 14. For comparison, the result for the MPPA with honeycomb backing cavity of the same size at the same position \((W=0.18\text{ m})\) is also provided. It can be seen that, in this particular case, the MPPA with an empty backing cavity outperforms the one with honeycomb in suppressing the targeted resonance peak. The control effect, however, largely depends on the location of the MPPA (see the curve with \(W=0.36\) m). In order to explain this phenomenon, eigenvalue analyses similar to that performed in Section 5.1 is conducted. The equivalent modal damping brought about by the MPPA to a particular enclosure mode can be described by the loss factor quantified by the imaginary part of the generalized eigenvalues of the coupled system. Fig. 15 shows the plot of the effective loss factor against the location of the MPPA \(W\). Indeed, \(W=0.18\) m corresponds to the maximum loss factor, resulting in a significant reduction of the resonance peak shown in Fig. 14. On the contrary, the loss factor reaches minimum when \(W=0.36\) m, corresponding to a slightly damped resonance peak in Fig. 14.

The location-dependent damping effect of MPPA is further explored by examining the spatial coupling between the acoustic fields across the MPP. Using Eq. (16), the free vibration equation for the \(m\)th mode of the enclosure coupled with the MPPA, after suppressing the source term on the right-hand side of the equation, can be written as

\[
\left(k_1^2 - k^2\right) + j k C_{\text{MPP}} \sum_m \frac{L_{m,m}^{(1)}}{\Lambda_{1m} A_m} - j k C_{\text{MPP}} \sum_n \frac{R_{m,n} B_n}{\Lambda_{1m} A_m} = 0.
\]  

\[
(32)
\]

Fig. 14. Space-averaged quadratic pressure for the enclosure without MPPA and partially covered by MPPA.
The final summation term in Eq. (32), combined with Eqs. (9) and (20), can be rewritten as

\[ R_{\text{norm}} = \int_{S_a} \frac{\phi_m}{A_m} p_d dS_a = \mathcal{R} e^{i\Theta}. \] (33)

In the vicinity of the targeted resonant frequency, where the acoustic field inside the enclosure is dominated by one enclosure mode \( m \), \( R_{\text{norm}} \) is in fact a normalized measure of the spatial waveform matching between the targeted enclosure mode and the wave field inside MPPA backing cavity. It can be referred to as spatial matching coefficient. Eq. (33) suggests that the spatial matching across MPP surface is quantified by the parameters \( \mathcal{R} \) and \( \Theta \), which represent the degree of spatial similarity between the two waveforms and the relative phase of the waveforms, respectively. Using the same configuration, \( \mathcal{R} \) and \( |\Theta| \) are plotted against \( W \) in Fig. 16. As expected, the position of the MPPA has a considerable influence on the wave matching across the MPP surface. In particular, when \( W = 0.18 \) m, the two waveforms have a rather strong but out-of-phase spatial matching \( (\mathcal{R} = 0.75, \ |\Theta| = 135^\circ) \). This situation can be roughly called out-of-phase spatial matching, which results in effective sound absorption. This phenomenon can be better observed in Fig. 17, in which the acoustic pressure distributions in the coupled system for the two \( W \) values are plotted at the resonant frequencies of the damped peaks, that is, 1554 and 1546 Hz, respectively. It can be seen that, for \( W = 0.18 \) m, the wave patterns (across MPP) on the two sides of MPP are similar and tend to be out of phase, whereas for \( W = 0.36 \) m, they are different but rather in phase.

The analyses described above demonstrate the possibility of controlling a particular resonance peak of the enclosure using the lateral modes of the MPPA backing cavity, which can be an alternative to the locally reactive type MPPA (such as honeycomb backing) that fails to suppress certain resonances. The normalized spatial matching coefficient \( R_{\text{norm}} \), together with the MPP-specific acoustic admittance \( C_{\text{MPP}} \), governs the effective loss factor that can possibly be introduced to the
targeted mode of the enclosure. The former depends on the location and geometry of the MPPA and the latter is controlled by the physical property of the MPP. Thus, for constructing an effective design of an MPPA containing an entire backing cavity, an out-of-phase wave matching with suitable frequency properties of the MPP should be considered.

6. Conclusions

The in situ sound absorption properties of MPPAs with two backing configurations are investigated in this paper: one contains an entire air volume and the other has a honeycomb structure at the back. Although the standard impedance tube measurements predict very similar sound absorption coefficient curves, they in fact exhibit drastically different in situ sound absorption behaviors when placed inside a compact acoustic enclosure. For the former configuration, an effective coupling between the backing cavity and the enclosure arises via lateral modes, which weakens or even neutralizes the energy dissipation capacity of the MPP. As a result, broadband sound absorption derived from the impedance tube measurement could not be materialized. Numerical and experimental studies show that an MPPA should be considered as part of the entire acoustic system rather than being treated as a locally reactive absorption boundary. The inner partitions of the honeycomb backing structure destroy the lateral modes formed in the backing cavity, thus yielding a local response of the MPPA to the acoustic loading upon its surface. This leads to superior in situ sound absorption inside the enclosure when one enclosure wall is fully covered by an MPPA. When partially covered, the backing cavity of the MPPA as well as its location can be designed to cope with a particular cavity resonance. A properly designed MPPA with suitable frequency properties should be placed on the enclosure wall in such a way that an out-of-phase spatial waveform matching between the two acoustic fields is generated, leading to a maximum pressure difference across the MPP panel. In this case, optimal use of the effect of the lateral modes of the MPPA backing cavity can be obtained, which outperforms its honeycomb counterpart in attenuating particular enclosure resonances.

In conclusion, this study shows that an effective design and an accurate prediction of the in situ sound absorption of MPPs inside compact acoustic enclosures require meticulous considerations of the backing configuration as well as its coupling with the front enclosure. The study suggests that MPPA should be treated as an integral part of the system, rather than a sound absorption boundary characterized by the surface impedance, calculated or measured in simple acoustic environment. The selection of the MPPA backing configuration depends on practical needs. For broadband noise control, honeycomb backing might be a suitable solution; however, for narrow-band resonant noise control, a well-designed volume-type MPPA might be a better option.

Acknowledgments

The authors wish to acknowledge the grant from the Research Grants Council of Hong Kong Special Administrative Region, China (Grants PolyU 5103/13E). Cheng Yang is grateful to Mr. Kening Yang for his assistance during the experiment.
References