### AMA1501 Introduction to Statistics for Business
### Miscellaneous Questions Set 6 Outline Suggested Solution

1. (a)

<table>
<thead>
<tr>
<th>Class mark (x)</th>
<th>Frequency, f</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>7</td>
</tr>
<tr>
<td>300</td>
<td>12</td>
</tr>
<tr>
<td>500</td>
<td>19</td>
</tr>
<tr>
<td>700</td>
<td>26</td>
</tr>
<tr>
<td>900</td>
<td>18</td>
</tr>
<tr>
<td>1250</td>
<td>10</td>
</tr>
<tr>
<td>1750</td>
<td>5</td>
</tr>
<tr>
<td>2500</td>
<td>3</td>
</tr>
</tbody>
</table>

\[ \sum f = 100 \quad \sum fx = 76950 \quad \sum f x^2 = 82907500 \]

Mean = \[ \frac{76950}{100} = \$769.50 \]

Standard deviation = \[ \sqrt{\frac{100(82907500) - 76950^2}{100(100-1)}} = \$489.22 \]

Mode = \[ 600 + \frac{26-19}{(26-19) + (26-18)}(800-600) = \$693\frac{1}{3} \]

(b)

<table>
<thead>
<tr>
<th>Amount less than ($'000)</th>
<th>Cumulative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>7</td>
</tr>
<tr>
<td>400</td>
<td>19</td>
</tr>
<tr>
<td>600</td>
<td>38</td>
</tr>
<tr>
<td>800</td>
<td>64</td>
</tr>
<tr>
<td>1000</td>
<td>82</td>
</tr>
<tr>
<td>1500</td>
<td>92</td>
</tr>
<tr>
<td>2000</td>
<td>97</td>
</tr>
<tr>
<td>3000</td>
<td>100</td>
</tr>
</tbody>
</table>
\( D_y = 1000 + \frac{90 - 82}{10} (1500 - 1000) = $1400 \)

(c) Let \( y \) be the amount of reimbursement.

\( y: 100 \ 300 \ 500 \ 700 \ 900 \ 1000 \ 1000 \ 1000 \ 1500 \ 1000 \ $1400 \)

\[ \sum f_y = 66200 \ \sum f_y^2 = 51220000 \]

Mean = \( \frac{66200}{100} = $662 \)

Standard deviation = \( \sqrt{\frac{100(51220000) - 66200^2}{100(100 - 1)}} = $273.32 \)

(d) \( \hat{p} = \left( \frac{1000 - 900}{1000 - 800} \times 18 + 10 + 5 + 3 \right) / 100 = 0.27 \)

Let \( X \) be the number of receipts have the amount greater than $900.

\( X \sim B(5, 0.27) \)

\[ \Pr(X \leq 2) = \sum_{x=0}^{2} C_x \left( 0.27 \right)^x \left( 0.73 \right)^{5-x} = 0.8743 \]

2.

(a) (i) \( \Pr (\text{Job satisfaction} \text{ is not the most important factor}) \)

\[ = 1 - \Pr (\text{Job satisfaction} \text{ is the most important factor}) \]

\[ = 1 - \frac{6!}{7!} = \frac{6}{7} \]

(ii) \( \Pr (\text{Job satisfaction} \text{ is most important and Working environment is least important}) \)

\[ = \frac{1!5!!}{7!} = \frac{1}{42} \]

(b) A: graduate is a Bachelor

B: graduate is employed

\( \Pr(A) = 0.8 \ \ \Pr(B) = 0.4 \ \ \Pr(B|A) = 0.3 \)

(i) \( \Pr(A \cap \overline{B}) = 1 - [0.8 + 0.4 - 0.8 \times 0.3] = 0.04 \)

(ii) \( \Pr(A|\overline{B}) = \frac{\Pr(A) - \Pr(A \cap B)}{1 - \Pr(B)} = \frac{0.8 - 0.8 \times 0.3}{1 - 0.4} = \frac{14}{15} \)
(c) A – a batch of bottle is supplied by Factory A
B – a batch of bottle is supplied by Factory B
C – a batch of bottle is supplied by Factory C
D – a batch has one broken bottle
\[ \Pr(A) = 0.55 \quad \Pr(B) = 0.3 \quad \Pr(C) = 0.15 \]
\[ \Pr(D|A) = \binom{10}{1} (0.05)^1 (0.95)^9 = 0.3151 \]
\[ \Pr(D|B) = \binom{10}{1} (0.08)^1 (0.92)^9 = 0.3777 \]
\[ \Pr(D|C) = \binom{10}{1} (0.1)^1 (0.9)^9 = 0.3874 \]
\[ \Pr(A|D) = \frac{0.55 \times 0.3151}{0.55 \times 0.3151 + 0.3 \times 0.3777 + 0.15 \times 0.3874} = 0.5027 \]

3. (a) \( X \) – delivery time (minutes), \( X \sim N(45, 8^2) \)

(i) \( \Pr(X \leq 43) = \Pr(Z \leq -0.25) = 0.4013 \)

(ii) Let \( k \) by the required maximum delivery time
\[ \Pr(X < k) = \Pr \left( Z < \frac{k - 45}{8} \right) = 0.9 \Rightarrow \frac{k - 45}{8} = 1.282 \Rightarrow k = 55.25 \text{ minutes} \]

(iii) \( \bar{X} \sim N(45, 8^2/10) \)
\[ \Pr(40 < \bar{X} < 47) \approx \Pr(-1.98 < Z < 0.79) = 1 - 0.0239 - 0.2148 = 0.7613 \]

(b) \( X \) – number of students purchase the textbook
\( X \sim B(180, 0.8) \)
Since \( n > 30, np > 5, nq > 5 \) and \( 0.1 < p < 0.9 \), normal approximation is used.
\[ \mu = 180 \times 0.8 = 144, \sigma^2 = 180 \times 0.8 \times 0.2 = 28.8 \]
\[ \Pr(X \geq 140) = \Pr(X > 139.5) \approx \Pr(Z > -0.84) = 1 - 0.2005 = 0.7995 \]

(c) \( X \) – number of products sold in 1 hour
\( X \sim Po(6) \)
\[ \Pr(5 \leq X \leq 7) = \sum_{x=5}^{7} \frac{e^{-6} 6^x}{x!} = 0.16062 + 0.16062 + 0.13768 = 0.4592 \]
\( Y \) – number of hours having hourly sales of 5 – 7 products
Y~B(10, 0.45892)

\[ \Pr(X = 5) = \binom{10}{5} (0.45892)^5 (1-0.45892)^5 = 0.2379 \]

4. (a) \( \bar{x} = \frac{21.4}{10} = 2.14 \) minutes, \( s = \sqrt{\frac{10(49.08) - 21.4^2}{10(10-1)}} = 0.60406 \) minutes

A 95% confidence interval for mean completion time is

\[ 2.14 \pm 2.262 \times \frac{0.60406}{\sqrt{10}}, \text{i.e., } 1.7079 < \mu < 2.5721 \) (minutes)

(b) \( H_0 : p_1 = p_2 \)

\( H_1 : p_1 < p_2 \)

\( \alpha = 0.025 \)

Critical region: \( z < -1.96 \)

\[ \hat{p}_1 = 0.48 \quad \hat{p}_2 = 0.72 \quad \hat{p} = \frac{240 + 360}{500 + 500} = 0.6 \]

Under \( H_0 \), test statistic

\[ z = \frac{(0.48 - 0.72) - 0}{\sqrt{0.6(0.4)\left(\frac{1}{500} + \frac{1}{500}\right)}} = -7.746 \]

Decision: Reject \( H_0 \)

(c) \( D = \) price quoted by supplier A – price quoted by supplier B

d: 100 -5 0 50 100 50 25 -20 10 100

\[ \bar{d} = \frac{410}{10} = 41, \quad s = \sqrt{\frac{10(36150) - 410^2}{10(10-1)}} = 46.3561 \]

\( H_0 : \mu_d = 0 \)

\( H_1 : \mu_d > 0 \)

\( \alpha = 0.05 \)

Critical region: \( t > 1.833 \)

Under \( H_0 \), test statistic

\[ t = \frac{41 - 0}{46.3561/\sqrt{10}} = 2.7969 \]

Decision: Reject \( H_0 \)
5. (a) 

$H_0 : \mu = 8000$

$H_1 : \mu < 8000$

$\alpha = 0.01$

Critical region: $t < -2.821$

Under $H_0$, test statistic 

$$t = \frac{6500 - 8000}{1200/\sqrt{10}} = -3.953$$

Decision: Reject $H_0$

(b) 

$H_0 : \text{number of complaints received in an hour follows Poisson distribution}$

$H_1 : H_0 \text{ is false}$

$\alpha = 0.05$

$$\bar{x} = \frac{103}{100} = 1.03$$

<table>
<thead>
<tr>
<th>No. of complaints</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>$\geq 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_i$</td>
<td>33</td>
<td>40</td>
<td>19</td>
<td>7</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$E_i$</td>
<td>35.7</td>
<td>36.77</td>
<td>18.94</td>
<td>6.50</td>
<td>1.67</td>
<td>0.41</td>
</tr>
<tr>
<td>$E_i$</td>
<td>35.7</td>
<td>36.77</td>
<td>18.94</td>
<td>8.59</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Critical region: $\chi^2 > 5.991, \nu = 2$

Under $H_0$, test statistic

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 0.5285$$

Decision: Do not reject $H_0$

(c) $H_0 : \text{air quality and temperature are independent}$

$H_1 : \text{air quality and temperature are not independent}$

$\alpha = 0.01$

Critical region: $\chi^2 > 13.277, \nu = 4$

Expected frequencies:
<table>
<thead>
<tr>
<th>Temperature</th>
<th>Air quality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below average</td>
<td>Poor 3.5</td>
</tr>
<tr>
<td>Average</td>
<td>Poor 14.5</td>
</tr>
<tr>
<td>Above average</td>
<td>Poor 7.0</td>
</tr>
</tbody>
</table>

Under $H_0$, test statistic $\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = 10.789$

**Decision:** Do not reject $H_0$

6. (a) (i) 

\[
b = \frac{8(31731.7) - (4644)(46.2)}{8(3208996) - (4644)^2} = 0.009573
\]

\[
a = \frac{46.2}{8} - 0.009573 \times \frac{4644}{8} = 0.217674
\]

\[
\hat{y} = 0.217674 + 0.009573x
\]

(ii) When $x = 700$, \( \hat{y} = 0.217674 + 0.009573 \times 700 = 6.919015 \times 10^6 \text{ Btu} \)

(b) (i) Sales = 34.1046 + 3.7459(Budget) – 30.0463(Ratio) + 0.0859(Income)

(ii) 

\[
a = 29657.75375 - 28741.07878 = 916.67497
\]

\[
b = 3
\]

\[
c = 14 - 3 - 1 = 10
\]

\[
d = 14 - 1 = 13
\]

\[
e = 28741.07878/3 = 9580.36
\]

\[
f = 916.67497/10 = 91.6675
\]

\[
g = 9580.36/91.6675 = 104.51
\]

(iii) \( 1 - R^2 = 1 - \frac{28741.07878}{29657.75375} = 0.0309 \)
(iv) $H_0: \beta_1 = \beta_2 = \beta_3 = 0$

$H_1$: at least one $\beta_i \neq 0, i = 1, 2, 3$

$\alpha = 0.05$

Critical region: $f > 3.71$

Under $H_0$, test statistic $f = 104.51$

Decision: Reject $H_0$